Superembedding Approach to

超嵌入方法:

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Abstract

摘要

We review a geometrical, so-called superembedding, approach to the description of the dynamics of point-like and extended supersymmetric objects (superbranes) in string theory. The approach is based on a supersymmetric extension of the classical surface theory to the description of superbrane dynamics by means of embedding worldvolume supersurfaces into target superspaces. Lorentz harmonics, twistors, and pure spinors are its intrinsic ingredients. The main new results obtained with this approach include the following ones. Being manifestly doubly supersymmetric (on the worldvolume and in target superspace), the superembedding approach explained that the local fermionic kappa-symmetry of the Green-Schwarz-like superbrane actions originates from the conventional local supersymmetry of the worldvolume. It established or clarified a classical relationship between various formulations of the dynamics of superparticles and superstrings, such as the Neveu-Schwarz-Ramond and the Green-Schwarz formulation. The full set of the equations of motion of the M-theory five-brane was first derived with the use of this approach.

我们综述弦理论中描述点状与延展超对称物体 (超膜) 动力学的几何方法，即所谓的超嵌入方法。该方法基于经典曲面理论的超对称推广，通过将世界体积超曲面嵌入目标超空间来描述超膜动力学。洛伦兹调和、扭量和纯旋量是其固有组成部分。利用该方法得到的主要新成果如下: 超嵌入方法具有显式双超对称性 (世界体积和目标超空间均具有超对称性)，它阐明了格林-施瓦茨类超膜作用量的局域费米子卡帕对称性来源于世界体积的常规局域超对称性。它建立并厘清了超粒子、超弦动力学的不同表述 (如纳维-施瓦茨-拉蒙德表述和格林-施瓦茨表述) 之间的经典关系。M 理论五膜的完整运动方程组正是利用该方法首次推导得到的。

Keywords

关键词

String theory - Supergravity - Supersymmetric p-branes - Superspace - Superfields - Worldvolume geometry and symmetries - Twistors - Pure spinors - Lorentz harmonics - Spinor moving frame

弦理论-超引力-超对称 p 膜-超空间-超场-世界 volume 几何与对称性-扭量-纯旋量-洛伦兹调和-旋量活动标架

Introduction

引言

This chapter is intended to give an introduction into a geometrical, so-called superembedding, approach to the description of the dynamics of supersymmetric particles and extended supersymmetric objects such as strings, membranes, and higher-dimensional branes. These objects are part of string theory where they play an important role in the perturbative and non-perturbative structure of the theory.

本章旨在介绍一种几何化的超嵌入方法，用于描述超对称粒子以及弦、膜和更高维膜这类延展超对称对象的动力学。这些对象都是弦论的组成部分，在该理论的微扰和非微扰结构中都发挥着重要作用。

We will call these dynamical supersymmetric objects super-p-branes, where p stands for the dimensionality of the object, so that p = 0 means a particle, p = 1 means a string, $p = 2$ means a membrane, and so on. Among these there are Dirichlet p-branes emerged as Dirichlet boundaries of open strings [1-5], an M2-brane [6,7], and an M5-brane [8-11] which live in an 11-dimensional corner extending string theory to M-theory, and there are also more exotic objects such as Neveu-Schwarz 5-branes, Kaluza-Klein monopoles, and some other (see [12-17] for more details and references).

我们将这些动力学超对称对象称为超 p-膜，其中 p 代表该对象的维度，因此 p = 0 对应粒子，p = 1 对应弦，$p = 2$ 对应膜，依此类推。其中包括作为开弦狄利克雷边界出现的狄利克雷 p-膜 [1-5]，存在于将弦论推广为 M 理论的 11 维角落中的 M2 膜 [6,7] 和 M5 膜 [8-11]，还有更多奇特的对象，例如纳维-施瓦茨 5 膜、卡鲁扎-克莱因磁单极子等等 (更多细节和参考文献见 [12-17])。

The worldvolume dynamics of all (or almost all) of these objects in a spacetime enlarged with Grassmann-odd spinorial directions (i.e., in target superspace) is described by a well-known formulation which we shall conventionally call the Green-Schwarz (GS) formulation constructed for superstrings in [18, 19], though for different branes it has been developed by different people. Let us particularly mention a 3-brane action in a $D = 6$ superspace constructed by Hughes, Liu, and Polchinski [20] and the 11D supermembrane action of Bergshoeff, Sezgin, and Townsend [6,7].

所有 (或几乎所有) 这类对象在扩充了格拉斯曼奇自旋方向的时空 (即目标超空间) 中的世界体积动力学，都由一个广为人知的框架描述，我们习惯上将其称为格林-施瓦茨 (GS) 框架，该框架最初是为超弦构建的 [18,19]，不过不同的膜对应的 GS 框架是由不同研究者发展出来的。特别要提及的是 Hughes、Liu 和 Polchinski 在 $D = 6$ 超空间中构建的 3 膜作用量 [20]，以及 Bergshoeff、Sezgin 和 Townsend 构建的 11 维超膜作用量 [6,7]。

For the superstrings there exists an alternative formulation, which was historically the first one, known under the name of Neveu, Schwarz, and Ramond, the NSR (fermionic or spinning) string [21-24]. Analogous formulation exists for spinning particles [25, 26] , while it seems not possible to construct in the same way spinning membranes or higher-dimensional branes [27] (As in the case of the ”Green-Schwarz” (GS) formulation, we will use the alias ”NSR” for the spinning particles and the spinning strings just for short.).

超弦还存在另一种框架，它出现的时间更早，名为纳维-施瓦茨-拉蒙德框架，即 NSR(费米或旋转) 弦 [21-24]。对于旋转粒子 [25, 26] 也存在类似的框架，但似乎无法用同样的方式构建旋转膜或更高维膜 [27](和“格林-施瓦茨”(GS) 框架的情况一样，我们使用“NSR”作为旋转粒子和旋转弦的简称。)

The GS and the NSR formulations are very well known, but there are also less conventional formulations of superbrane dynamics such as Lorentz-harmonic formulations [28-46] and, in particular cases of superparticles and superstrings, twistor formulations [47-56].

GS 框架和 NSR 框架都广为人知，但超膜动力学还有一些非主流框架，例如洛伦兹调和框架 [28-46]，在超粒子和超弦的特殊情况下还有扭量框架 [47-56]。

The Lorentz-harmonic approach, especially in its form developed in [32,35,36, 38,40,41], which is also known under the name of spinor moving frame formalism, is actually closely related to the twistor approach. It provided a natural basis for a higher-dimensional generalization of the twistor formulation [45,46,57]. In addition to the "standard" superparticles, superstrings, and superbranes, in [58] and [59,60], the spinor moving frame formalism was also applied to the twistor string [61, 62] and the ambitwistor string [63, 64] . This allowed one to establish the relation of the 4-dimensional twistor and ambitwistor strings to a so-called null superstring (see [65, 66] and references therein) and to generalize the twistor string to higher dimensions [58] and the ambitwistor string to 11 (and 4) dimensions [59]. The Lorentz-harmonic approach was also applied to the study of amplitudes of $D = 10$ supersymmetric gauge theory and D = 11 supergravity [67-69].

洛伦兹调和方法，尤其是在 [32,35,36, 38,40,41] 中发展出的形式，也被称为自旋动框架式，它实际上和扭量方法密切相关。它为扭量 formulation 的高维推广提供了自然基础 [45,46,57]。除了“标准的”超粒子、超弦和超膜，自旋动框架式还在 [58] 以及 [59,60] 中被应用于扭量弦 [61,62] 和双扭量弦 [63, 64] 。这让我们得以建立 4 维扭量弦、双扭量弦与所谓零超弦的联系 (见 [65,66] 及其中的参考文献)，还将扭量弦推广到了更高维 [58]，将双扭量弦推广到了 11(及 4) 维 [59]。洛伦兹调和方法还被用于研究 $D = 10$ 超对称规范理论和 D = 11 超引力的振幅 [67-69]。

The twistor and the Lorentz-harmonic formulations were proposed as an attempt to solve the problem of the covariant quantization of the superparticles and superstrings. But though interesting and important results have been obtained using these methods (especially in the case of superparticles), the problem of the covariant quantization of the GS superstring remained and became even more topical in view of the interest in studying string theory in nontrivial supergravity backgrounds with nonzero antisymmetric gauge fields of the Ramond-Ramond type, such as anti-de Sitter (AdS) spaces, pp-waves, and string compactifications with Ramond-Ramon fluxes for which the NSR formulation is not applicable.

扭量框架和洛伦兹调和框架的提出，都是为了解决超粒子和超弦的协变量子化问题。尽管利用这些方法已经得到了许多有趣且重要的结果 (尤其是超粒子领域)，但 GS 超弦的协变量子化问题仍未解决，甚至变得更加紧迫: 目前人们越来越关注非平凡超引力背景下、带有非零拉蒙德-拉蒙德型反对称规范场的弦论，例如反德西特 (AdS) 空间、pp 波，以及带有拉蒙德-拉蒙德通量的弦紧致化，而 NSR 框架并不适用于这些情形。

The superembedding approach also arose from the desire to solve the problem of the covariant quantization of the Green-Schwarz superstring. The idea was to find a more general (doubly supersymmetric) formulation which would encompass the main positive features of the NSR, GS, and twistor formulation, thus allowing one to make a progress in quantizing the superstring covariantly. Such a formulation was first realized for superparticles and then for superstrings, and in the end for all known super-p-branes by efforts of several theoretical groups [52,53,70-96]. In [89] the spinor moving frame variables were incorporated into the doubly super-symmetric approach, and this gave rise to the present form of the superembedding formalism.

超嵌入方法也源于解决格林-施瓦茨超弦协变量子化问题的需求。其核心思想是找到一个更具普遍性的 (双超对称) 表述，囊括 NSR 表述、GS 表述和扭量表述的主要优点，从而推动超弦协变量子化的研究进展。多个理论研究团队的努力下，这种表述先后在超粒子、超弦中实现，最终推广到所有已知的超 p 膜 [52,53,70-96]。文献 [89] 将旋量活动标架变量引入双超对称方法，形成了如今形式的超嵌入形式体系。

The progress in the covariant quantization of the superstring during the last decades has been mainly made by Berkovits [74,75,79,87]. In 2000 he put forward a powerful quantization prescription [97, 98] based on the use of pure spinors (see, e.g., [99-104] for its developments and [105] for the most recent review and a detailed list of references). Geometrical roots of these techniques stem from the superembedding approach [106]. The relation of the pure spinor approach and the spinor moving frame approach in the cases of an $11D$ superparticle and a $10D$ superstring was discussed in [45, 46] and [107] .

过去几十年间，超弦协变量子化的主要进展由 Berkovits 完成 [74,75,79,87]。他在 2000 年提出了基于纯旋量使用的强有力量子化方案 [97, 98] (相关发展可参见例如 [99-104]，最新综述和详细参考文献列表见 [105])。这些技术的几何根源可追溯到超嵌入方法 [106]。纯旋量方法与旋量活动标架方法在 $11D$ 超粒子和 $10D$ 超弦情形下的关系，已在 [45, 46] 和 [107] 中讨论。

The superembedding approach is an elegant, profound, and in a sense universal geometrical formulation, and the purpose of this chapter is to explain its main features (see [108, 109] for detailed reviews).

超嵌入方法是一种优雅、深刻且在某种意义上具有普适性的几何表述，本章的目的是讲解它的主要特征 (详细综述参见 [108, 109])。

The superembedding approach is based on a supersymmetric extension of the classical surface theory and its application to bosonic relativistic strings [110,111] to the description of superbrane dynamics by means of embedding worldvolume supersurfaces into target superspaces [89]. Being manifestly doubly supersymmetric (on the worldvolume and in target superspace), the superembedding approach has explained the origin and the nature of a local fermionic symmetry (so-called κ -symmetry) of the GS formulation as a manifestation of the conventional local supersymmetry of the worldvolume. In this way it has solved the problem of infinite reducibility of the κ -symmetry by realizing it as an irreducible extended worldvolume supersymmetry [70]. As we have mentioned, this stimulated progress in the covariant quantization of the Green-Schwarz superstring.

超嵌入方法基于经典曲面理论的超对称推广，将经典曲面理论对玻色相对论弦的应用延伸到: 通过世界体超曲面嵌入目标超空间，描述超膜动力学 [89]。超嵌入方法具有明显的双超对称性 (世界体和目标超空间都具备超对称性)，它阐明了 GS 表述中局域费米对称 (即所谓的 κ 对称) 的起源与本质，指出该对称是世界体常规局域超对称性的体现。通过将该对称实现为不可约的扩展世界体超对称性，它解决了 κ 对称的无限可约性问题 [70]。正如我们前文提到，这推动了格林-施瓦茨超弦协变量子化的进展。

The superembedding approach established or clarified a classical relationship between various formulations of the dynamics of superparticles and superstrings, such as the NSR and the GS formulation [71, 72, 82, 85, 95] , the twistor, and harmonic descriptions.

超嵌入方法确立并厘清了超粒子、超弦动力学不同表述 (如 NSR 表述、GS 表述 [71, 72, 82, 85, 95] 、扭量表述和调和描述) 之间的经典关系。

This approach has proved to be a universal and powerful method applicable to the description of all known supersymmetric branes, in particular, to those of them for which standard methods encountered problems because of their specific structure, such as the 5-brane of M-theory. The superembedding methods allowed, for the first time, to derive the complete set of covariant equations of motion of the M5-brane [91, 92]. And only later these equations were obtained [112] from the M5-brane action [113, 114] based on a different technique adapted to deal with duality-symmetric and self-dual (chiral) p-form fields [115, 116]. An alternative super-5-brane action was proposed in [117]. However, the self-duality of the field strength of the 2-form gauge field on the M5-brane worldvolume does not follow from this action. Nevertheless, if imposed separately, it is consistent with the equations of motion of the model.

该方法已被证明是一种通用且强大的工具，可用于描述所有已知的超对称膜，尤其适用于那些因特殊结构导致标准方法遇到困难的超膜，例如 M 理论的 5 膜。超嵌入方法首次推导出了 M5 膜完整的协变运动方程组 [91, 92]。这些方程之后才在另一项研究 [112] 中从 M5 膜作用量 [113, 114] 得到，该研究基于一套不同的技术，专门用于处理对偶对称和自对偶 (手征)p 形式场 [115, 116]。文献 [117] 提出了另一种超 5 膜作用量，但 M5 膜世界体上 2 形式规范场场强的自对偶性无法从该作用量导出；不过若单独施加该条件，它与模型的运动方程是自洽的。

The superembedding formulation has also proved to be useful for studying the "brany" mechanism of partial supersymmetry breaking [20, 118-121] by giving a geometrical recipe [122-124] for the construction of covariant world-volume supersymmetric actions for superbranes. Upon gauge fixing worldvolume superdiffeomorphisms, these actions become those of effective field theories with nonlinearly realized spontaneously broken supersymmetries. This has demonstrated an intrinsic link of the superembedding approach and the method of nonlinear realizations put forward in application to supersymmetric theories in [125-133] (see [134, 135] for a review and further references).

超嵌入表述也被证明可用于研究部分超对称性破缺的“膜”机制 [20, 118-121]，它为构造超膜的协变世界体超对称作用量提供了几何方案 [122-124]。固定世界体超微分同胚规范后，这些作用量就成为具有非线性实现的自发破缺超对称性的有效场论作用量。这表明超嵌入方法与 [125-133] 中应用于超对称理论的非线性实现方法存在内在联系 (综述及更多参考文献参见 [134, 135])。

We now pass to a more detailed consideration of general properties of this geometrical formulation starting from the description of its ingredients.

现在我们从介绍该几何表述的基本组成开始，进一步详细讨论它的一般性质。

Worldvolume vs Target Space Supersymmetry

世界体积 vs 目标空间超对称

The "Neveu-Schwarz-Ramond" Formulation

“内沃-施瓦茨-拉蒙德”表述

In the NSR formulation which describes spinning particles [25,26] and spinning strings [21-24], as well as recently proposed ambitwistor string in its original formulation [63], the worldline or worldsheet of the spinning object is a supersurface $\mathcal{M}_{sw}$ parametrized by bosonic coordinates ξ^m and fermionic coordinates η^μ which we will collectively call $z^M = (\xi^m, \eta^\mu)$. Depending on the model considered, $\mathcal{M}_{sw}$ may have a various number of fermionic directions η . The dynamics of the spinning object is described by embedding $\mathcal{M}_{sw}$ into a bosonic target space-time M_{TS} parametrized by coordinates $X^{\underline{m}}\left(\underline{m} = 0, 1, \cdots, D-1\right)$ (To avoid the proliferation of the indices, in what follows we will denote by Latin indices the components of the bosonic directions and by Greek indices the components of fermionic directions of the worldvolume superspace. The underlined Latin and Greek indices are associated with target superspace directions, while (underlined) capital Latin indices stand for both bosonic and fermionic directions.). In the classical problems, the number of space-time dimensions can be arbitrary, but for quantum consistency the spinning string, whose worldsheet has one or two fermionic directions, must live in a ten-dimensional target space.

在描述自旋粒子 [25,26]、自旋弦 [21-24] 以及近期提出的原表述下的振幅弦 [63] 的 NSR 表述中，自旋物体的世界线或世界面是超曲面 $\mathcal{M}_{sw}$ ，由玻色坐标 ξ^m 和费米坐标 η^μ 参数化，我们将其统称为 $z^M = (\xi^m, \eta^\mu)$ 。根据所考虑的模型，$\mathcal{M}_{sw}$ 可拥有不同数量的费米方向 η 。自旋物体的动力学由将 $\mathcal{M}_{sw}$ 嵌入坐标 $X^{\underline{m}}\left(\underline{m} = 0, 1, \cdots, D-1\right)$ 参数化的玻色目标时空 M_{TS} 描述 (为避免指标过多，下文我们用拉丁指标表示世界体积超空间玻色方向的分量，用希腊指标表示世界体积超空间费米方向的分量。带下划线的拉丁、希腊指标对应目标超空间方向，而 (带下划线的) 大写拉丁指标同时表示玻色方向和费米方向。)。经典问题中，时空维度数可以任意，但为满足量子自洽性，世界面含一或两个费米方向的自旋弦必须存在于十维目标空间中。

The motion of the spinning object is described by the image of $\mathcal{M}_{sw}$ in M_{TS} :

自旋物体的运动由 $\mathcal{M}_{sw}$ 在 M_{TS} 中的像描述:

$$X^{\underline{m}}\left(z^M\right) = x^{\underline{m}}(\xi) + i\eta\chi^{\underline{m}}(\xi), \tag{1}$$

where, for simplicity, we assumed that $\mathcal{M}_{sw}$ has the single Grassmann-odd direction parametrized by $\eta, x^{\underline{m}}(\xi)$ is associated with the bosonic degrees of freedom of the spinning particle or string in M_{TS} , and the Grassmann-odd vector $\chi^{\underline{m}}(\xi)$ is associated with its spin degrees of freedom.

其中，为简化起见，我们假设 $\mathcal{M}_{sw}$ 仅有一个由 $\eta, x^{\underline{m}}(\xi)$ 参数化的格拉斯曼奇方向，$\eta, x^{\underline{m}}(\xi)$ 对应 M_{TS} 中自旋粒子或弦的玻色自由度，格拉斯曼奇矢量 $\chi^{\underline{m}}(\xi)$ 对应其自旋自由度。

The NSR formulation is invariant under worldsheet superdiffeomorphisms $z^M \to z'^M\left(z^M\right)$, which include bosonic reparametrizations of $\mathcal{M}_w$:

NSR 表述在世界面超微分同胚 $z^M \to z'^M\left(z^M\right)$ 下不变，该变换包含 $\mathcal{M}_w$ 的玻色重新参数化:

$$\delta\xi = a(\xi), \tag{2}$$

and local worldsheet supersymmetry

以及局部世界面超对称

$$\delta\eta = \kappa(\xi)\,,\ \delta\xi = i\kappa(\xi)\,\eta. \tag{3}$$

The presence of the local symmetries (2) and (3) implies that the dynamics of the spinning objects is subject to bosonic and fermionic first-class constraints, respectively, which we will schematically write down in the form:

对称性 (2) 和 (3) 的存在说明，自旋物体的动力学分别受玻型和费米型第一类约束，我们将其概要地写为如下形式:

$$(\partial x^{\underline{m}})^2 = 0,\ \partial x^{\underline{m}}\chi_{\underline{m}} = 0. \tag{4}$$

The bosonic constraint in (4) stands for the mass shell or the Virasoro conditions, and the fermionic constraint produces, upon quantization, the Dirac equation for the spin wave functions of the dynamical system.

(4) 中的玻色约束对应质壳条件或维罗拉索条件，费米约束经量子化后，给出动力学系统自旋波函数的狄拉克方程。

The NSR formulation does not have target-space supersymmetry. The latter appears, in the case of the spinning string, only at the quantum level upon imposing the Gliozzi-Scherk-Olive projection [136]. A merit of this formulation is that it is covariantly quantizable [137].

NSR 表述不具备目标空间超对称性。对于自旋弦, 该对称性仅在量子层面通过施加格里奥齐-舍克-奥利弗投影后出现 [136]。该表述的优点在于它可以协变量子化 [137]。

The ”Green-Schwarz” Formulation

「格林-施瓦茨」表述

This formulation is applicable to all known superparticles, superstrings, and conventional superbranes. Now the worldvolume $\mathcal{M}_w$ is a (p+1)-dimensional bosonic surface parametrized by the coordinates ξ^m $(m = 0, 1, \cdots, p)$ and the target space, into which $\mathcal{M}_w$ is embedded, is a superspace M_{TS} parametrized by bosonic coordinates $x^{\underline{m}}\left(\underline{m} = 0, 1, \cdots, D-1\right)$ and by an appropriate number of fermionic coordinates $\theta^{\underline{\alpha}}\left(\underline{\alpha} = 1, \cdots, 2n\right)$

该表述适用于所有已知的超粒子、超弦和常规超膜。此时世界 volume $\mathcal{M}_w$ 是一个由坐标 ξ^m $(m = 0, 1, \cdots, p)$ 参数化的 (p+1) 维玻恩茨曲面，$\mathcal{M}_w$ 嵌入的目标空间是由玻恩茨坐标 $x^{\underline{m}}\left(\underline{m} = 0, 1, \cdots, D-1\right)$ 和适量费米子坐标 $\theta^{\underline{\alpha}}\left(\underline{\alpha} = 1, \cdots, 2n\right)$ 参数化的超空间 M_{TS}

$$Z^{\underline{M}}(\xi) = \left(x^{\underline{m}}(\xi), \theta^{\underline{\alpha}}(\xi)\right). \tag{5}$$

The GS formulation is manifestly invariant under bosonic reparametrizations $\xi \to \xi'(\xi)$ of the worldvolume $\mathcal{M}_w$, and target space superdiffeomorphisms $Z^{\underline{M}} \to Z'^{\underline{M}}(Z^{\underline{M}})$, which in the case of flat target superspace reduce to the translations along $x^{\underline{m}}$ and to the global target-space supersymmetry transformations

GS 表述在世界 volume $\mathcal{M}_w$ 的玻恩茨重参数化 $\xi \to \xi'(\xi)$ 以及目标空间超微分同胚 $Z^{\underline{M}} \to Z'^{\underline{M}}(Z^{\underline{M}})$ 下具有明显不变性；在平直目标超空间的情况下，这些不变性退化为沿 $x^{\underline{m}}$ 的平移和全局目标空间超对称变换

$$\delta\theta^{\underline{\alpha}} = \varepsilon^{\underline{\alpha}},\ \delta x^{\underline{m}} = i\bar{\theta}\Gamma^{\underline{m}}\delta\theta. \tag{6}$$

There is also another (non-manifest) local worldvolume fermionic symmetry, so-called κ -symmetry, inherent to the GS formulation. This is an important symmetry which implies and reflects the existence of supersymmetric BPS brane-like solutions of corresponding supergravity theories. It is thus responsible for the "brane scan" (i.e., it prescribes which brane lives in which target superspace). The κ -symmetry was first observed in the case of superparticles [138, 139] and then became an important ingredient of the Green-Schwarz-like actions for the superstring [18,19], the supermembrane [6, 7] , the Dp-branes [140-144], and the M5-brane [113,114].

GS 表述还固有另一种非明显的局域世界 volume 费米对称性，即所谓的 κ 对称性。这一重要对称性预示并反映了对应超引力理论中存在超对称 BPS 类膜解，因此它是「膜扫描」的核心 (即它规定了哪类膜存在于哪类目标超空间中)。κ 对称性最早在超粒子的情况中被发现 [138, 139]，之后成为超弦 [18,19]、超膜 [6, 7] 、Dp 膜 [140-144] 和 M5 膜 [113,114] 的格林-施瓦茨类作用量的重要组成部分。

Kappa-symmetry transformations have the following generic form (in flat target superspace):

卡帕对称性变换具有如下通用形式 (在平直目标超空间中):

$$\delta\theta^{\underline{\alpha}} = \Pi^{\underline{\alpha}}{}_{\underline{\beta}}\kappa^{\underline{\beta}}(\xi),\ \delta x^{\underline{m}} = -i\bar{\theta}\Gamma^{\underline{m}}\delta\theta, \tag{7}$$

where $\Pi^{\alpha}{}_{\beta}$, in the most of the cases, is a half-rank projector matrix, whose form depends on the object under consideration. Because of the presence of the projector in the κ -transformations, only half of $\kappa^{\underline{\alpha}}$, i.e., n of the $2n$ Grassmann spinor components, effectively contribute to the variation of the worldvolume fields. That is, the κ -symmetry is (actually infinitely) reducible. However, without the use of some auxiliary fields, it is not possible to single out the independent components of the κ -symmetry transformations in a Lorentz-covariant way. This causes the problem of the covariant quantization of the GS formulation.

其中在大多数情况下，$\Pi^{\alpha}{}_{\beta}$ 是一个半秩投影矩阵，其形式由研究对象决定。由于 κ 变换中存在投影器，仅一半的 $\kappa^{\underline{\alpha}}$ ，即 $2n$ 格拉斯曼旋量分量中的 n ，能有效影响世界 volume 场的变分。也就是说，κ 对称性实际上是可约的 (且为无限可约)。但如果不借助辅助场，就无法以洛伦兹协变的方式分离出 κ 对称性变换的独立分量，这正是 GS 表述协变量子化问题的来源。

To solve the problem of infinite reducibility of the κ -symmetry, one should try to find its irreducible realization which is covariant in target superspace. A natural assumption is that this should be an extended local worldvolume supersymmetry (3) with the number of independent parameters equal to the number of independent (irreducible) κ -symmetries [70]. This reasoning brings us to the doubly supersym-metric formulation.

为了解决 κ 对称性无限可约的问题，我们需要寻找它在目标超空间中协变的不可约实现。一个自然的假设是，它应为扩展局域世界 volume 超对称性 (3)，其独立参数的数量等于独立 (不可约) κ 对称性的数量 [70]。这一思路将我们引向双超对称表述。

The Doubly Supersymmetric Formulation

双超对称表述

In this formulation the dynamics of superbranes is described by embedding a worldvolume supersurface $\mathcal{M}_{sw}$ parametrized by the coordinates $z^M=(\xi^m,\eta^\alpha)$ $(m=0,1,\cdots,p)$, $(\alpha=1,\cdots,n)$ into a target superspace M_{TS} parametrized by the coordinates $Z^{\underline{M}}=(X^{\underline{m}},\Theta^{\underline{\alpha}})\big(\underline{m}=0,1,\cdots,D-1\big),\big(\underline{\alpha}=1,\ldots,2n\big)$. Note that the number of the Grassmann directions of $\mathcal{M}_{sw}$ is half the number of the Grassmann directions of M_{TS}. Such a choice of the supermanifolds for superembedding is caused by our desire to identify n local supersymmetries on $\mathcal{M}_{sw}$ with n -independent κ -symmetries of the GS formulation. But for some applications [71-73, 76, 77, 85], in particular, as far as quantization is concerned [75,79,87,106], one may also consider an embedding of supersurfaces $\mathcal{M}_{sw}$ with less number of Grassmann directions. In these cases models can contain residual non-manifest κ -symmetry.

在该表述中，超膜动力学由坐标 $z^M=(\xi^m,\eta^\alpha)$ $(m=0,1,\cdots,p)$, $(\alpha=1,\cdots,n)$ 参数化的世界体积超曲面 $\mathcal{M}_{sw}$ 嵌入坐标 $Z^{\underline{M}}=(X^{\underline{m}},\Theta^{\underline{\alpha}})\big(\underline{m}=0,1,\cdots,D-1\big),\big(\underline{\alpha}=1,\ldots,2n\big)$ 参数化的目标超空间 M_{TS} 来描述。注意 $\mathcal{M}_{sw}$ 的格拉斯曼方向数是 M_{TS} 格拉斯曼方向数的一半。这种对超嵌入超流形的选择，源于我们希望将 $\mathcal{M}_{sw}$ 上的 n 局域超对称与 GS 表述中独立于 n 的 κ 对称对应起来。但对于部分应用 [71-73, 76, 77, 85]，尤其是涉及量子化的场景 [75,79,87,106]，也可以考虑格拉斯曼方向数更少的超曲面 $\mathcal{M}_{sw}$ 嵌入。在这些情况下，模型可包含残余非显式 κ 对称。

Thus in the doubly supersymmetric formulation, the degrees of freedom of the superbranes are described by worldvolume superfields:

因此在双超对称表述中，超膜的自由度由世界体积超场描述:

$$X^{\underline{m}}\left(z^M\right)=x^{\underline{m}}(\xi)+i\eta^\alpha\chi_\alpha^{\underline{m}}(\xi)+\cdots,\ \Theta^{\underline{\alpha}}\left(z^M\right)=\theta^{\underline{\alpha}}(\xi)+\eta^\alpha\lambda_\alpha^{\underline{\alpha}}(\xi)+\cdots,$$

(8)

where $\cdots$ stand for the terms of higher order in η^α. These terms contain auxiliary fields and in addition, for example, in the case of the D-branes and the M5-brane, include the gauge fields propagating on the worldvolumes of these branes.

其中 $\cdots$ 代表 η^α 中的高阶项。这些项包含辅助场，此外例如在 D 膜和 M5 膜的情况中，还包含在这些膜的世界体积上传播的规范场。

From Eq. (8) we see that in the doubly supersymmetric construction, the number of degrees of freedom of the superbrane roughly speaking doubles. We now have $x^{\underline{m}}(\xi)$ describing the bosonic oscillations of the brane, the Grassmann "spin"- vectors $\chi_{\alpha}^{m}(\xi)$ as in the NSR formulation, the Green-Schwarz fermionic spinor degrees of freedom $\theta^{\underline{\alpha}}(\xi)$, and their bosonic counterparts $\lambda_{\alpha}^{\underline{\alpha}}(\xi)$. So if all these worldvolume fields are independent, the corresponding models will not describe conventional superbranes. In this case one will get, for instance, so-called spinning superparticles and spinning superstrings [145-147] which have more degrees of freedom than the conventional NSR and GS dynamical systems. In addition to local worldvolume supersymmetry, they also have an infinite reducible κ -symmetry as an independent local fermionic symmetry.

从式 (8) 可以看出，在双超对称构造中，超膜的自由度数量粗略而言加倍了。我们现在有描述膜玻色振荡的 $x^{\underline{m}}(\xi)$ ，类似 NSR 表述的格拉斯曼“自旋”矢量 $\chi_{\alpha}^{m}(\xi)$ ，格林-施瓦茨费米子自旋自由度 $\theta^{\underline{\alpha}}(\xi)$ ，以及它们的玻色对应量 $\lambda_{\alpha}^{\underline{\alpha}}(\xi)$ 。因此如果所有这些世界体积场都是独立的，对应的模型将无法描述传统超膜。这种情况下会得到所谓的旋转超粒子和旋转超弦 [145-147]，它们比传统 NSR 和 GS 动力学系统拥有更多自由度。除世界体积局域超对称外，它们还拥有作为独立局域费米对称的无限可约 κ 对称。

To reach our goal of interpreting κ -symmetry as a manifestation of the local worldvolume supersymmetry, we should find an appropriate doubly supersymmetric description of the conventional superbranes. For this we should impose constraints on the superfields (8) which relate their components in such a way that the independent physical degrees of freedom described by these superfields will correspond to the standard GS formulation. The geometrical meaning of these constraints is that they cause the worldvolume supersurface $\mathcal{M}_{sw}$ to be imbedded into the target superspace M_{TS} in a specific way.

为了将 κ 对称诠释为世界体积局域超对称的表现，我们需要找到传统超膜的恰当双超对称描述。为此我们需要对超场 (8) 施加约束，让约束关联超场各分量，使得这些超场描述的独立物理自由度与标准 GS 表述一致。这些约束的几何意义是，它们要求世界体积超曲面 $\mathcal{M}_{sw}$ 以特定方式嵌入目标超空间 M_{TS} 。

The Geometrical Meaning of the Superembedding Condition

超嵌入条件的几何意义

The generic requirement for the superembedding to be appropriate to the description of the dynamics of the superparticles, superstrings, and super-p-branes is as follows. Let us consider a supersurface $\mathcal{M}_{sw}$ (to be associated with the worldvolume of a super-p-brane), whose geometry is described by supervielbein one-forms $e^{a}\left(z^{M}\right)\ (a=0,1,\cdots,p)$ and $e^{\alpha}\left(z^{M}\right)(\alpha=1,\cdots n)$, and a curved target superspace M_{TS} , whose geometry is described by supervielbein one-forms $E^{\underline{a}}\left(Z^{\underline{M}}\right)(\underline{a}=\ 0,1,\cdots,D-1)$ and $E^{\underline{\alpha}}\left(Z^{\underline{M}}\right)\left(\underline{\alpha}=1,\cdots 2n\right)$ (Note that the supergeometries of $\mathcal{M}_{sw}$ and M_{TS} should be that of corresponding supergravities, which implies that the torsions and curvatures of $\mathcal{M}_{sw}$ and M_{TS} are subject to appropriate supergravity constraints.).

超嵌入适合描述超粒子、超弦和超 p 膜动力学的通用要求如下。我们考虑一张超曲面 $\mathcal{M}_{sw}$ (对应超 p 膜的世界体积)，其几何由超标架一元形 $e^{a}\left(z^{M}\right)\ (a=0,1,\cdots,p)$ 和 $e^{\alpha}\left(z^{M}\right)(\alpha=1,\cdots n)$ 描述；再考虑一个弯曲目标超空间 M_{TS} ，其几何由超标架一元形 $E^{\underline{a}}\left(Z^{\underline{M}}\right)(\underline{a}=\ 0,1,\cdots,D-1)$ 和 $E^{\underline{\alpha}}\left(Z^{\underline{M}}\right)\left(\underline{\alpha}=1,\cdots 2n\right)$ 描述 (请注意，$\mathcal{M}_{sw}$ 和 M_{TS} 的超几何应为对应超引力的超几何，即 $\mathcal{M}_{sw}$ 和 M_{TS} 的挠率与曲率满足恰当的超引力约束)。

The superembedding construction is manifestly covariant under superdiffeomor-phism transformations of the superworldvolume coordinates:

超嵌入构造在超世界体积坐标的超微分同胚变换下是明显协变的:

$$z^{M} \rightarrow z'^{M}(z) \tag{9}$$

In the cases of superparticles and superstrings, this local superworldvolume symmetry can be reduced to a suitable (superconformal) subgroup.

对于超粒子和超弦，该局域超世界体积对称性可以约化为恰当的 (超共形) 子群。

For the superembedding of $\mathcal{M}_{sw}$ into M_{TS} to describe a super-p-brane propagating in M_{TS} , the superworldvolume pullback of $E^{\underline{a}}$ along the Grassmann directions of $\mathcal{M}_{sw}$ must vanish, i.e., in

要让 $\mathcal{M}_{sw}$ 到 M_{TS} 的超嵌入描述在 M_{TS} 中传播的超 p 膜，$E^{\underline{a}}$ 沿 $\mathcal{M}_{sw}$ 格拉斯曼方向的超世界体积拉回必须为零，即

$$E^{\underline{a}}(Z(z)) = e^{a} E_{a}^{\underline{a}} + e^{\alpha} E_{\alpha}^{\underline{a}} \tag{10}$$

the Grassmann components are zero

格拉斯曼分量为零

$$E_{\alpha}^{\frac{a}{2}}(Z(z)) = 0. \tag{11}$$

For instance, in the case of a flat target superspace (which, for simplicity, we will mostly consider in this review), a supersymmetric vector supervielbein is given by a Volkov-Akulov one-form:

例如，在平坦目标超空间的情形 (为简化起见，本综述大部分内容都考虑该情形)，对称向量超标架由沃尔科夫-阿库洛夫一元形给出:

$$E^{\underline{a}} = dX^{\underline{a}} - i d\overline{\Theta} \Gamma^{\underline{a}} \Theta. \tag{12}$$

Then the superembedding condition (11) means that the components of the pullback of (12) along e^{α} vanish:

此时超嵌入条件 (11) 表明，(12) 沿 e^{α} 的拉回分量为零:

$$\nabla_{\alpha} X^{\underline{a}} - i \nabla_{\alpha} \overline{\Theta} \Gamma^{\underline{a}} \Theta = 0, \tag{13}$$

where ∇_{α} are fermionic covariant derivatives in the (generically curved) superworld-volume $\mathcal{M}_{sw}$.

其中 ∇_{α} 是 (一般弯曲的) 超世界体积 $\mathcal{M}_{sw}$ 中的费米子协变导数。

For the most of the superbranes (with some subtleties for the space filling and codimension one branes [123, 124, 148, 149]), the superembedding condition (11), accompanied by the M_{TS} and/or $\mathcal{M}_{sw}$ supergravity constraints, implies that:

对于大多数超膜 (填充分数维膜和余维 1 膜存在一些细微之处 [123, 124, 148, 149])，结合 M_{TS} 和/或 $\mathcal{M}_{sw}$ 超引力约束的超嵌入条件 (11) 可以推出:

- the geometry of the superworldvolume $\mathcal{M}_{sw}$ is induced by its embedding into M_{TS} , i.e., the $\mathcal{M}_{sw}$ supergravity on the brane is not propagating;

- 超世界体积 $\mathcal{M}_{sw}$ 的几何由其到 M_{TS} 的嵌入诱导，也就是说，膜上的 $\mathcal{M}_{sw}$ 超引力不传播；

- the dynamics of the superbrane is subject to the standard constraints of the Green-Schwarz formulation, such as the Virasoro constraints and their fermionic counterparts;

- 超膜动力学满足格林-施瓦茨表述的标准约束，例如 Virasoro 约束及其费米对应约束；

- κ -symmetry is a particular form of worldvolume superdiffeomorphisms;

- κ 对称性是世界体积超微分同胚的一种特殊形式；

- the consistency of the superembedding condition results in the same ”brane scan” as that of the Green-Schwarz formulation.

- 超嵌入条件的自洽性给出的” 膜扫描” 结果与格林-施瓦茨表述完全一致。

In addition, when the number of the Grassmann directions of $\mathcal{M}_{sw}$ is 16 (or higher), the integrability of the superembedding condition requires the worldvolume superfields to satisfy the dynamical equations of motion of the superbrane [88- 90] (This was a reason to the name ”geometrodynamic equation” used for the superembedding condition in [86, 89] . Note also that this is similar to the case of maximally supersymmetric super-Yang-Mills and supergravity theories in $D \geq 3$ whose superfield constraints produce the dynamical equations of motion.). It is in this way that the covariant equations of motion of the M5-brane were obtained for the first time [91,92]. We will review the superembedding description of the M2- and M5-brane in section ”Superembedding Description of M2- and M5-Branes.”

此外，当 $\mathcal{M}_{sw}$ 的格拉斯曼维数为 16(或更高) 时，超嵌入条件的可积性要求世界体积超场满足超膜的动力学运动方程 [88-90](这就是 [86, 89] 中将超嵌入条件称为“几何动力学方程”的原因。还需注意，这一情况与 $D \geq 3$ 中最大超对称的超杨-米尔斯理论和超引力理论类似，这些理论的超场约束就给出了动力学运动方程)。M5 膜的协变运动方程正是通过这种方法首次得到的 [91,92]。我们将在“M2 膜与 M5 膜的超嵌入描述”一节中回顾 M2 膜与 M5 膜的超嵌入描述。

Superembedding Description of Superparticles

超粒子的超嵌入描述

$N = 1, D = 3$ Superparticle

$N = 1, D = 3$ 超粒子

Let us now consider the dynamical consequences of the superembedding condition in the simplest case of a superparticle propagating in a flat $N = 1, D = 3$ target superspace [70]. Then the supersurface $\mathcal{M}_{sw}$ of the previous subsection is associated with the superparticle "worldline" having one bosonic (time) and one fermionic coordinate (ξ, η) , and the target superspace is parametrized by bosonic three-vector coordinates $X^{\underline{m}}\left(\underline{m} = 0,1,2\right)$ and Grassmann-Majorana two-spinor coordinates $\Theta^{\underline{\alpha}}\left(\underline{\alpha} = 1,2\right)$. The superembedding condition (13) relates the worldline superfields $X^{\underline{m}}\left(z^M\right)$ and $\Theta^{\underline{\alpha}}\left(z^M\right)$ in the following way (for simplicity we assume the geometry of the target superspace be flat):

现在我们来讨论超粒子在平直 $N = 1, D = 3$ 目标超空间中传播这一最简单情形下，超嵌入条件的动力学推论 [70]。上一小节的超曲面 $\mathcal{M}_{sw}$ 对应超粒子的“世界线”，该世界线拥有 1 个玻色 (时间) 坐标和 1 个费米坐标 (ξ, η) ，目标超空间由玻色三维矢量坐标 $X^{\underline{m}}\left(\underline{m} = 0,1,2\right)$ 和格拉斯曼-马约拉纳双旋量坐标 $\Theta^{\underline{\alpha}}\left(\underline{\alpha} = 1,2\right)$ 参数化。超嵌入条件 (13) 将世界线超场 $X^{\underline{m}}\left(z^M\right)$ 和 $\Theta^{\underline{\alpha}}\left(z^M\right)$ 按下式关联 (为简化起见，我们假设目标超空间的几何是平直的):

$$DX^{\underline{m}} - iD\overline{\Theta}\Gamma^{\underline{m}}\Theta = 0, \tag{14}$$

where D is a Grassmann covariant derivative on $\mathcal{M}_{sw}$ which in the case of the superparticles can be chosen to be flat

其中 D 是 $\mathcal{M}_{sw}$ 上的格拉斯曼协变导数，对于超粒子该导数可选取为平直的

$$D = \frac{\partial}{\partial\eta} + i\eta\frac{\partial}{\partial\xi},\ \{D, D\} = 2i\partial_\xi. \tag{15}$$

Using the η -expansion (8), which in the case under consideration does not contain "..."-terms, we obtain the following relation between the components of the superfields $X^{\underline{m}}\left(z^M\right)$ and $\Theta^{\underline{\alpha}}\left(z^M\right)$:

利用不包含“省略项”的 η 展开式 (8)，我们得到超场分量 $X^{\underline{m}}\left(z^M\right)$ 和 $\Theta^{\underline{\alpha}}\left(z^M\right)$ 满足以下关系:

$$\partial_\xi x^{\underline{m}} - i\partial_\xi\overline{\theta}\Gamma^{\underline{m}}\theta = \overline{\lambda}\Gamma^{\underline{m}}\lambda, \tag{16}$$

$$\chi^{\underline{m}} = \overline{\theta}\Gamma^{\underline{m}}\lambda. \tag{17}$$

From Eq. (16) we see that $\lambda^{\underline{\alpha}}(\xi)$ are not independent fields but are expressed in terms of the derivatives of $x^{\underline{m}}$ and $\theta^{\underline{\alpha}}$. Moreover in the l.h.s. of (16), one can recognize the canonical momentum of the superparticle $P^{\underline{m}} = \frac{1}{e(\tau)}\left(\partial_\xi x^{\underline{m}} - i\partial_\xi\overline{\theta}\Gamma^{\underline{m}}\theta\right)$ (here $e(\xi)$ is a proportionality coefficient; its geometrical meaning is to be the worldline gravitational field.) whose square is identically zero:

从式 (16) 可以看出，$\lambda^{\underline{\alpha}}(\xi)$ 并非独立场，而是由 $x^{\underline{m}}$ 和 $\theta^{\underline{\alpha}}$ 的导数表示得到。此外，我们可以认出式 (16) 左侧就是超粒子 $P^{\underline{m}} = \frac{1}{e(\tau)}\left(\partial_\xi x^{\underline{m}} - i\partial_\xi\overline{\theta}\Gamma^{\underline{m}}\theta\right)$ 的正则动量 (其中 $e(\xi)$ 是比例系数，其几何意义是世界线引力场)，它的平方恒为零:

$$P^{\underline{m}}P_{\underline{m}} = 0 \tag{18}$$

due to its so-called Cartan-Penrose (or twistor) representation as a bilinear combination of commuting (twistor-like) spinor components λ and the Γ -matrix identities

这是因为它有所谓的嘉当-彭罗斯 (或扭量) 表示，即对易 (类扭量) 旋量分量 λ 的双线性组合，再结合 Γ 矩阵恒等式

$$\Gamma^{\underline{m}}_{\underline{\alpha\beta}}\Gamma_{\underline{m\gamma\delta}} + \Gamma^{\underline{m}}_{\underline{\beta\gamma}}\Gamma_{\underline{m\alpha\delta}} + \Gamma^{\underline{m}}_{\underline{\gamma\alpha}}\Gamma_{\underline{m\beta\delta}} = 0. \tag{19}$$

We conclude that the superparticle is massless.

我们的结论是该超粒子是无质量的。

In the case of the superstrings, the superembedding condition will produce in a similar way the Virasoro constraints, and it will produce the corresponding constraints for the other superbranes.

对于超弦，超嵌入条件会以类似的方式给出 Virasoro 约束，对其他超膜，它也会给出相应的约束。

Equation (17) implies the relation between the Grassmann vector and the Grassmann spinor variables, so that only one or another can be taken to describe independent fermionic degrees of freedom. This is a basic relation which allows one to establish a classical correspondence between the NSR and the GS formulation of supersymmetric particles and strings [71, 72, 82, 85, 95] .

方程 (17) 给出了格拉斯曼矢量与格拉斯曼旋量变量之间的关系，因此只能选取其中一类来描述独立的费米自由度。这是一个基础关系，它帮助我们建立超对称粒子与弦的 NSR 表述和 GS 表述之间的经典对应 [71, 72, 82, 85, 95] 。

Local Worldvolume Supersymmetry Versus κ -Symmetry

局部世界体超对称性对 κ 对称性

Let us now demonstrate how κ -symmetry appears in the superembedding formulation as a weird realization of the local worldvolume supersymmetry.

现在我们来演示 κ 对称性如何在超嵌入表述中作为局部世界体超对称性的一种特殊实现出现。

The components (7) of the superfields $X^{\underline{m}}(z^M)$ and $\Theta^{\underline{\alpha}}(z^M)$ transform under the local worldline supersymmetry (3) in the standard way:

超场 $X^{\underline{m}}(z^M)$ 和 $\Theta^{\underline{\alpha}}(z^M)$ 的分量 (7) 在局部世界线超对称性 (3) 下按标准方式变换:

$$\delta\theta^{\underline{\alpha}} = -\lambda^{\underline{\alpha}}\kappa(\xi), \; \delta\lambda^{\underline{\alpha}} = i\partial_{\xi}\theta^{\underline{\alpha}}\kappa(\xi), \tag{20}$$

$$\delta x^{\underline{m}} = i\chi^{\underline{m}}\kappa(\xi)\,,\ \delta\chi^{\underline{m}} = -\partial_\xi x^{\underline{m}}\kappa(\xi)\,. \tag{21}$$

We now substitute into the first equation of (21) the solution (17) of the superem-bedding condition (14) and observe that, due to the form of the θ -variation (20), the variation of $x^{\underline{m}}$ can be rewritten as follows:

现在我们将超嵌入条件 (14) 的解 (17) 代入 (21) 的第一个方程，可以看到，由于 θ 变分 (20) 的形式，$x^{\underline{m}}$ 的变分可以改写为如下形式:

$$\delta x^{\underline{m}} = i\left(\bar{\theta}\Gamma^{\underline{m}}\lambda\right)\kappa(\xi) = -i\bar{\theta}\Gamma^{\underline{m}}\delta\theta. \tag{22}$$

The next step is to replace the Grassmann scalar parameter of the local supersymmetry by the scalar product of the spinor λ_β with a Grassmann spinor parameter $\kappa\underline{\beta}(\xi)$, which is always possible:

下一步我们将局部超对称性的格拉斯曼标量参数替换为旋量 λ_β 与格拉斯曼旋量参数 $\kappa\underline{\beta}(\xi)$ 的标量积，这总是可行的:

$$\kappa(\xi) = 2\lambda_{\underline{\beta}}\kappa^{\underline{\beta}}(\xi)\,, \tag{23}$$

and to substitute (23) into the θ -variation (20). We thus get:

再将 (23) 代入 θ 变分 (20)，我们得到:

$$\delta\theta^{\underline{\alpha}} = -2\lambda^{\underline{\alpha}}\lambda_{\underline{\beta}}\kappa^{\underline{\beta}}(\xi)\,. \tag{24}$$

We now note that, due to the superembedding condition (16), the bilinear combination of λ in (24) is nothing but

我们注意到，由于超嵌入条件 (16)，(24) 中 λ 的双线性组合正是

$$\Pi^{\alpha}_{\underline{\beta}} = -2\lambda^{\underline{\alpha}}\lambda_{\underline{\beta}} = \left(\partial_\xi x^{\underline{m}} - i\partial_\xi\bar{\theta}\Gamma^{\underline{m}}\theta\right)\left(\Gamma_{\underline{m}}\right)^{\alpha}_{\underline{\beta}}, \tag{25}$$

which is the projector matrix in the κ -symmetry variation of θ (7). Hence, the local supersymmetry variations (22) and (24) reduce to the κ -variations (7).

它就是 θ 的 κ 对称性变分 (7) 中的投影矩阵。因此，局部超对称性变分 (22) 和 (24) 约化为 κ 变分 (7)。

We have thus demonstrated how, in virtue of the superembedding condition (14), the κ -symmetry of the GS formulation of the superbranes arises from the irreducible local worldvolume supersymmetry.

我们由此证明了，借助超嵌入条件 (14)，超膜 GS 表述中的 κ 对称性如何起源于不可约局部世界体超对称性。

One might have already noticed the difference in sign in the target-space super-symmetry variations of $x^{\underline{m}}$ (6) and in the worldvolume supersymmetry variations (22) and corresponding κ -variations (7). The rigid target-space supersymmetry and the local worldvolume supersymmetry (or κ -symmetry) can be therefore regarded as, respectively,"left" and (a part of) "right" supertranslations of $x^{\underline{m}}$.

读者可能已经注意到，$x^{\underline{m}}$ 的目标空间超对称性变分 (6) 与世界体超对称性变分 (22) 以及对应的 κ 变分 (7) 存在符号差异。因此，刚性目标空间超对称性和局部世界体超对称性 (或 κ 对称性) 可以分别看作是 $x^{\underline{m}}$ 的“左”超平移和 (一部分)“右”超平移。

The Worldline Superfield Action for the $N=1, D=3$ Superparticle

$N=1, D=3$ 超粒子的世界线超场作用量

The worldline $n=1$ superfield action, which produces the superembedding relations (14), (16), and (17), and also the superparticle equations of motion, has the following form:

给出超嵌入关系 (14)、(16)、(17) 以及超粒子运动方程的世界线 $n=1$ 超场作用量形式如下:

$$S=-i\int d\xi d\eta P_{\underline{m}}\left(DX^{\underline{m}}-iD\bar{\Theta}\Gamma^{\underline{m}}\Theta\right), \tag{26}$$

where $X^{\underline{m}}(\xi,\eta)$ and $\Theta^{\underline{\alpha}}(\xi,\eta)$ are the superfields defined in (8) and

其中 $X^{\underline{m}}(\xi,\eta)$ 和 $\Theta^{\underline{\alpha}}(\xi,\eta)$ 是式 (8) 中定义的超场，

$$P_{\underline{m}}(\xi,\eta)=p_{\underline{m}}(\xi)+i\eta\rho_{\underline{m}}(\xi) \tag{27}$$

is the Lagrange multiplier superfield.

是拉格朗日乘子超场。

Varying (26) with respect to $P_{\underline{m}}$, we get the superembedding condition (14), and hence (16) and (17).

对 (26) 关于 $P_{\underline{m}}$ 变分，我们得到超嵌入条件 (14)，进而得到 (16) 和 (17)。

To see what kind of other equations follow from the action, let us integrate it over η using the Berezin rules $\int d\eta=0, \int d\eta\eta=1$. As a result we get the following action for the components of the superfields (8) and (27):

为了说明该作用量还会给出哪些其他方程，我们利用贝雷津规则 $\int d\eta=0, \int d\eta\eta=1$ 对 η 积分，最终得到超场分量 (8) 和 (27) 的如下作用量:

$$S=\int d\xi p_{\underline{m}}\left(\partial_\xi x^{\underline{m}}-i\partial_\xi\bar{\theta}\Gamma^{\underline{m}}\theta-\bar{\lambda}\Gamma^{\underline{m}}\lambda\right)+i\int d\xi\rho_{\underline{m}}\left(\chi^{\underline{m}}-\bar{\lambda}\Gamma^{\underline{m}}\theta\right). \tag{28}$$

We see that p_m is the particle momentum and that the second term in (28) means that $\rho_{\underline{a}}$ and $\chi^{\underline{m}}$ are auxiliary fields satisfying algebraic equations:

我们可以看到，p_m 是粒子动量，(28) 中的第二项表明 $\rho_{\underline{a}}$ 和 $\chi^{\underline{m}}$ 是满足代数方程的辅助场:

$$\rho_{\underline{m}} = 0, \ \chi^{\underline{m}} = \bar{\lambda}\Gamma^{\underline{m}}\theta, \tag{29}$$

the latter being just part of the superembedding condition. Another part of the superembedding condition, namely, Eq. (16), is obtained from (28) as a variation of $p_{\underline{m}}$.

后者正是超嵌入条件的一部分。超嵌入条件的另一部分，即式 (16)，可由 (28) 对 $p_{\underline{m}}$ 变分得到。

The equations of motion of $x^{\underline{m}}$ and $\theta^{\underline{\alpha}}$ are, respectively,

$x^{\underline{m}}$ 和 $\theta^{\underline{\alpha}}$ 的运动方程分别为

$$\partial_\xi p_{\underline{m}} = 0 \ \text{ and } \ \left(p_{\underline{m}}\Gamma^{\underline{m}}\right)^{\underline{\alpha}}\partial_\xi\theta^{\underline{\beta}} = 0. \tag{30}$$

These are the standard equations of motion of a superparticle [139]. What remains to show is that the particle under consideration is massless, i.e., that its momentum is light-like $p^{\underline{m}}p_{\underline{m}} = 0$. To see this one should consider the equation of motion of the commuting spinor variable $\lambda^{\underline{\alpha}}$:

这些是超粒子的标准运动方程 [139]。余下需要证明的是, 该粒子是无质量的, 即其动量类光 $p^{\underline{m}}p_{\underline{m}} = 0$ 。为此我们需要考虑对易旋量变量 $\lambda^{\underline{\alpha}}$ 的运动方程:

$$\left(p_{\underline{m}}\Gamma^{\underline{m}}\right)^{\underline{\alpha}}{}_{\underline{\beta}}\lambda^{\underline{\beta}} = 0. \tag{31}$$

The general solution of this equation in $D = 3$, as well as in $D = 4, 6$ and 10 is

该方程在 $D = 3$ 以及 $D = 4, 6$ 和 10 中的通解为

$$p_{\underline{m}} = e(\xi)\,\bar{\lambda}\Gamma_{\underline{m}}\lambda = e(\xi)\left(\partial_\xi x^{\underline{m}} - i\partial_\xi\bar{\theta}\Gamma^{\underline{m}}\theta\right), \tag{32}$$

where $e(\xi)$ is an arbitrary worldline function, and the last equality being the consequence of (16). Hence $p^{\underline{m}}p_{\underline{m}} = 0$ in virtue of the Dirac matrix identity (19), and the particle is indeed massless.

其中 $e(\xi)$ 是任意世界线函数，最后一个等式是 (16) 的推论。因此结合狄拉克矩阵恒等式 (19) 可得 $p^{\underline{m}}p_{\underline{m}} = 0$ 成立，该粒子确实是无质量的。

On the other hand, (32) implies that

另一方面，(32) 可推出

$$\bar{\lambda}\Gamma_{\underline{m}}\lambda = \frac{p_{\underline{m}}}{e(\xi)}. \tag{33}$$

So, if we substitute Eq. (29) and the expression (33) for the bilinear of λ into the action (28), it reduces to a standard first-order action for a massless superparticle:

因此，如果我们将式 (29) 和 λ 双线性项的表达式 (33) 代入作用量 (28)，它就退化为无质量超粒子的标准一阶作用量:

$$S = \int d\xi \left(p_{\underline{m}} \left(\partial_\xi x^{\underline{m}} - i\partial_\xi \bar{\theta} \Gamma^{\underline{m}} \theta \right) - e^{-1}(\xi)\, p_{\underline{m}} p^{\underline{m}} \right). \tag{34}$$

Let us now consider constraints on the canonical fermionic momenta $\pi_{\underline{\alpha}} = \frac{\delta S}{\delta \dot{\theta}^{\underline{\alpha}}}$ conjugate to $\theta^{\underline{\alpha}}$. They are

现在我们来讨论与 $\theta^{\underline{\alpha}}$ 共轭的正则费米动量 $\pi_{\underline{\alpha}} = \frac{\delta S}{\delta \dot{\theta}^{\underline{\alpha}}}$ 满足的约束，其形式为

$$D_{\underline{\alpha}} = \pi_{\underline{\alpha}} - i p_{\underline{m}} (\Gamma^{\underline{m}} \theta)_{\underline{\alpha}} = 0. \tag{35}$$

The Poisson brackets of these constraints are

这些约束的泊松括号为

$$\left\{ D_{\underline{\alpha}}, D_{\underline{\beta}} \right\} = -2i p_{\underline{m}} \Gamma^{\underline{m}}_{\underline{\alpha\beta}}. \tag{36}$$

Since the momentum p_m is constrained (32) to be light-like, the r.h.s. of (36) is a degenerate 2×2 matrix of rank one. This means that the fermionic constraints (35) are the mixture of a first- and a second-class constraint. In the conventional Green-Schwarz formulation, it is not possible to separate in (35) an irreducible set of the first-class fermionic constraints from the second-class ones in a Lorentz-covariant way, which causes the covariant quantization problem. The availability of auxiliary commuting spinor variables $\lambda^{\underline{\alpha}}$ in the superembedding formulation allows one to do so. One can convince oneself that the constraint $\lambda^{\underline{\alpha}} D_{\underline{\alpha}}$ obtained by projecting $D_{\underline{\alpha}}$ along $\lambda^{\underline{\alpha}}$ is the first class:

由于动量 p_m 受约束条件 (32) 限制为类光，(36) 的右侧是一个秩为 1 的退化 2×2 矩阵。这说明费米子约束 (35) 是第一类约束和第二类约束的混合。在传统的格林-施瓦茨表述中，无法以洛伦兹协变的方式从 (35) 的第二类约束中分离出不可约的第一类费米子约束集合，这就引发了协变量子化问题。超嵌入表述中存在辅助对易旋量变量 $\lambda^{\underline{\alpha}}$，使得我们可以完成这一分离。可以验证，将 $D_{\underline{\alpha}}$ 沿 $\lambda^{\underline{\alpha}}$ 投影得到的约束 $\lambda^{\underline{\alpha}} D_{\underline{\alpha}}$ 是第一类约束:

$$\left\{ \lambda^{\underline{\alpha}} D_{\underline{\alpha}}, \lambda^{\underline{\beta}} D_{\underline{\beta}} \right\} = -2i p_{\underline{m}} \lambda \Gamma^{\underline{m}} \lambda \simeq 0. \tag{37}$$

The r.h.s. of (37) is weakly zero in the Dirac sense due to (31).

由于 (31) 的存在，(37) 的右侧在狄拉克意义上弱等于零。

The first-class bosonic constraint $p_{\underline{m}} \lambda \Gamma^{\underline{m}} \lambda$ and the first-class fermionic constraint $\lambda^{\underline{\alpha}} D_{\underline{\alpha}}$ form the one-dimensional (worldline) $n = 1$ superalgebra and generate local worldline reparametrizations and supersymmetry transformations, the latter being the irreducible counterpart of the kappa-symmetry of the Green-Schwarz formulation, as we have discussed in section " $N = 1, D = 3$ Superparticle." This provides us with an algebraic ground for the covariant quantization of superparticles and superstrings. Actually, as was realized in

[74, 75, 79, 87], a version of the superembedding formulation with manifest $n=2$ worldsheet supersymmetry turns out to be the most appropriate for quantizing $D=10$ superparticles and superstrings. It is this version which gives rise to pure spinors and is in the origin of the covariant quantization procedure of [105]. So we shall now turn to the consideration of an $N=1, D=10$ superparticle with $n=2$ worldline supersymmetry.

第一类玻色约束 $p_{\underline{m}}\lambda\Gamma^{\underline{m}}\lambda$ 和第一类费米约束 $\lambda^{\underline{\alpha}}D_{\underline{\alpha}}$ 构成一维 (世界线) $n=1$ 超代数，生成局域世界线重参数化与超对称变换；正如我们在“$N=1, D=3$ 超粒子”一节讨论过的，这里的超对称变换是格林-施瓦茨表述中 κ 对称性的不可约对应形式。这为超粒子和超弦的协变量子化提供了代数基础。实际上，正如文献 [74,75,79,87] 中指出，具有显式 $n=2$ 世界面超对称性的超嵌入表述版本被证明最适合对 $D=10$ 超粒子和超弦进行量子化。正是这一版本产生了纯旋量，也是文献 [105] 中协变量子化方案的起源。因此我们接下来讨论具有 $n=2$ 世界线超对称性的 $N=1, D=10$ 超粒子。

The $N=1, D=10$ Superparticle with $n=2$ Worldline Supersymmetry and Pure Spinors

具有 $n=2$ 世界线超对称的 $N=1, D=10$ 纯自旋超粒子

We are now in a 10-dimensional flat superspace parametrized by 10 bosonic coordinates $X^{\underline{m}}\left(\underline{m}=0,1,\cdots,9\right)$ and by 16 real Majorana-Weyl Grassmann coordinates $\Theta^{\underline{\alpha}}\left(\underline{\alpha}=1,\cdots,16\right)$.

我们现在处于由 10 个玻色坐标 $X^{\underline{m}}\left(\underline{m}=0,1,\cdots,9\right)$ 和 16 个实马约拉纳-外尔格拉斯曼坐标 $\Theta^{\underline{\alpha}}\left(\underline{\alpha}=1,\cdots,16\right)$ 参数化的 10 维平坦超空间中。

The superparticle worldline is assumed to be a supersurface $\mathcal{M}_{sw}$ with one bosonic and two fermionic directions $\left(\xi,\eta,\overline{\eta}\right)$, where η and $\overline{\eta}$ are complex conjugate to each other. The worldline fermionic covariant derivatives

超粒子世界线被假定为超曲面 $\mathcal{M}_{sw}$，具有一个玻色方向和两个费米方向 $\left(\xi,\eta,\overline{\eta}\right)$，其中 η 与 $\overline{\eta}$ 互为复共轭。世界线费米协变导数

$$D=\frac{\partial}{\partial\eta}+i\overline{\eta}\frac{\partial}{\partial\xi},\ \bar{D}=\frac{\partial}{\partial\overline{\eta}}+i\eta\frac{\partial}{\partial\xi},$$

generate the worldline $n=2$ superalgebra

生成世界线 $n=2$ 超代数

$$D^2=0,\ \bar{D}^2=0,\ \{D,\bar{D}\}=2i\frac{\partial}{\partial\xi}. \tag{38}$$

The $n=2$ superfield form of the $N=1, D=10$ superparticle action is

$N=1, D=10$ 超粒子作用量的 $n=2$ 超场形式为

$$S=-i\int d\xi d\eta d\overline{\eta}\left[P_{\underline{m}}\left(DX^{\underline{m}}-iD\Theta\Gamma^{\underline{m}}\Theta\right)+\bar{P}_{\underline{m}}\left(\bar{D}X^{\underline{m}}-i\bar{D}\Theta\Gamma^{\underline{m}}\Theta\right)\right],$$

(39)

where $P_{\underline{m}}(\xi,\eta,\overline{\eta})$ and $\bar{P}_{\underline{m}}(\xi,\eta,\overline{\eta})$ are complex conjugate Lagrange multipliers, whose variation produces the superembedding conditions:

其中 $P_{\underline{m}}(\xi,\eta,\overline{\eta})$ 和 $\bar{P}_{\underline{m}}(\xi,\eta,\overline{\eta})$ 是互为复共轭的拉格朗日乘子，对其变分可得到超嵌入条件:

$$D X^{\underline{m}}-i D \Theta \Gamma^{\underline{a}} \Theta=0,\ \bar{D} X^{\underline{m}}-i \bar{D} \Theta \Gamma^{\underline{m}} \Theta=0 \tag{40}$$

to be satisfied by the superfields

需由超场满足

$$X^{\underline{m}}(\xi,\eta,\overline{\eta})=x^{\underline{m}}(\xi)+i\eta\chi^{\underline{m}}(\xi)+i\overline{\eta\chi}^{\underline{m}}(\xi)+\eta\overline{\eta}y^{\underline{m}}(\xi),$$

$$\Theta^{\underline{\alpha}}(\xi,\eta,\overline{\eta})=\theta^{\underline{\alpha}}(\xi)+\eta\lambda^{\underline{\alpha}}(\xi)+\overline{\eta}\overline{\lambda}^{\underline{\alpha}}(\xi)+\eta\overline{\eta}\sigma^{\underline{\alpha}}(\xi),$$

where in addition to the component fields already known to the reader, which obey the relations similar to (16) and (17), there appear auxiliary fields $y^{\underline{m}}(\xi)$ and $\sigma^{\underline{\alpha}}(\xi)$ which are expressed through other components and/or their derivatives via the superembedding conditions (40). We shall not present the explicit form of these expressions here but just show how the pure spinor conditions follow from (40). To this end let us hit the first expression in (40) by D and the second one by $\bar{D}$. In view of the $n=2$ superalgebra commutation relations (38), we have:

除了读者已知的、满足类似 (16) 和 (17) 关系的分量场之外，还出现了辅助场 $y^{\underline{m}}(\xi)$ 和 $\sigma^{\underline{\alpha}}(\xi)$，它们可通过超嵌入条件 (40) 由其他分量和/或其导数表示。我们在此不给出这些表达式的显式形式，仅展示纯自旋条件如何从 (40) 推导得到。为此，我们用 D 作用于 (40) 的第一个表达式，用 $\bar{D}$ 作用于第二个表达式。根据 $n=2$ 超代数的对易关系 (38)，我们得到:

$$D \Theta \Gamma^{\underline{m}} D \Theta=0,\ \bar{D} \Theta \Gamma^{\underline{m}} \bar{D} \Theta=0,$$

and hence

因此

$$\lambda \Gamma^{\underline{m}} \lambda=0,\ \overline{\lambda} \Gamma^{\underline{m}} \overline{\lambda}=0. \tag{41}$$

By definition complex commuting spinors satisfying Eq. (41) are called pure spinors.

根据定义，满足方程 (41) 的复对易自旋量被称为纯自旋量。

As in the previous case of the $N=1, D=3$ superparticle, one can convince oneself that the following fermionic constraints are of the first class:

和之前 $N=1, D=3$ 超粒子的情况一样，不难验证以下费米约束是第一类约束:

$$\lambda^{\underline{\alpha}} D_{\underline{\alpha}} = \lambda^{\underline{\alpha}} \left(\pi_{\underline{\alpha}} - i p_{\underline{m}} (\Gamma^{\underline{m}} \theta)_{\underline{\alpha}} \right) = 0, \tag{42}$$

$$\bar{\lambda}^{\underline{\alpha}} D_{\underline{\alpha}} = \bar{\lambda}^{\underline{\alpha}} \left(\pi_{\underline{\alpha}} - i p_{\underline{m}} (\Gamma^{\underline{m}} \theta)_{\underline{\alpha}} \right) = 0.$$

In the pure spinor approach to the covariant quantization [105], the spinors $\bar{\lambda}$ never appear, the spinors λ are ghosts, and the constraint $Q = \lambda^{\underline{\alpha}} D_{\underline{\alpha}}$ plays the role of the BRST charge whose cohomology has been shown to reproduce the correct physical spectrum of quantized superparticles and superstrings.

在协变量子化的纯自旋方法 [105] 中，自旋量 $\bar{\lambda}$ 从不出现，自旋量 λ 是鬼场，约束 $Q = \lambda^{\underline{\alpha}} D_{\underline{\alpha}}$ 扮演 BRST 荷的角色，已证明其上同调可重现量子化超粒子和超弦的正确物理谱。

We have thus shown that ingredients of the pure spinor quantization procedure appear in the superembedding formulation. A relation between the two approaches has been established in [106] with the example of the heterotic string. We refer the reader to this article for further details and pass to the superembedding description of an $N = 1, D = 10$ superparticle with $n = 8$ worldline supersymmetry.

我们由此证明了纯自旋量子化过程的各要素都出现在超嵌入表述中。两种方法的关联已在文献 [106] 中以杂化弦为例建立。读者可参阅该文章了解更多细节，接下来我们转向具有 $n = 8$ 世界线超对称的 $N = 1, D = 10$ 超粒子的超嵌入描述。

$N = 1, D = 10$ Superparticle with $n = 8$ Worldline Supersymmetry

$N = 1, D = 10$ 具有 $n = 8$ 世界线超对称的超粒子

Let us consider now the double supersymemtric formulation of an $N = 1, D = 10$ superparticle with a worldline superspace having $n = 8$ real Grassmann coordinates $\eta^q, q = 1, \ldots, 8$. The worldline fermionic covariant derivatives are collected in an octuplet of the $SO(8)$ symmetry group:

现在我们来讨论 $N = 1, D = 10$ 超粒子的双超对称 formulation，其世界线超空间具有 $n = 8$ 个实格拉斯曼坐标 $\eta^q, q = 1, \ldots, 8$ 。世界线费米协变导数被收集为 $SO(8)$ 对称群的一个八重态:

$$D_q = \frac{\partial}{\partial \eta^q} + i \eta^q \frac{\partial}{\partial \xi} \tag{43}$$

and generate the worldline $n = 8$ supersymmetry algebra

并且生成世界线 $n = 8$ 超对称代数

$$\{D_q, D_p\} = 2i \delta_{qp} \partial_\xi, \ \partial_\xi := \frac{\partial}{\partial \xi}. \tag{44}$$

These derivatives can be obtained by decomposing the external differential in worldline superspace:

这些导数可通过分解世界线超空间中的外微分得到:

$$d = d\xi\partial_\xi + d\eta^q\partial_q = e\partial_\xi + d\eta^q D_q,\ e = d\xi - id\eta^q\eta^q. \tag{45}$$

in the basis of the bosonic and fermionic supervirlbeins

在玻色型和费米型超标架的基底下

$$e^A = (e, e^q) = (d\xi - id\eta^q\eta^q, d\eta^q)\,. \tag{46}$$

The bosonic and fermionic coordinate functions are now superfields with nine terms in the decomposition:

玻色坐标函数与费米坐标函数现在是超场，其分解包含九项:

$$X^{\underline{m}}(\xi,\eta) = x^{\underline{m}}(\xi) + i\eta^q\chi_q^{\underline{m}}(\xi) + \ldots + \frac{1}{8!}\eta^{q_8}\ldots\eta^{q_1}\varepsilon_{q_1\ldots q_8}y^{\underline{m}}(\xi)\,, \tag{47}$$

$$\Theta^{\underline{\alpha}}(\xi,\eta,\overline{\eta}) = \theta^{\underline{\alpha}}(\xi) + \eta^q\lambda_q^{\underline{\alpha}}(\xi) + \ldots + \frac{1}{8!}\eta^{q_8}\ldots\eta^{q_1}\varepsilon_{q_1\ldots q_8}\zeta^{\underline{\alpha}}(\xi)\,, \tag{48}$$

The superembedding equation has the form:

超嵌入方程的形式为:

$$E_q^{\underline{m}} = D_qX^{\underline{m}} - iD_q\Theta\Gamma^{\underline{m}}\Theta = D_qX^{\underline{m}} - iD_q\Theta^{\underline{\alpha}}\Gamma^{\underline{m}}_{\underline{\alpha\beta}}\Theta^{\underline{\beta}} = 0, \tag{49}$$

where the notation E_q^m reflects the fact that the l.h.s. of the superembedding condition is given by the fermionic component of the superworldline pullback of the $N = 1, D = 10$ Volkov-Akulov one-form:

其中记号 E_q^m 反映了如下事实: 超嵌入条件的左侧由 $N = 1, D = 10$ 沃尔科夫-阿库洛夫一形式拉回至超世界线后的费米分量给出:

$$E^{\underline{m}} = dX^{\underline{m}} - id\Theta\Gamma^{\underline{m}}\Theta, \tag{50}$$

where now $\Gamma^{\underline{m}}_{\alpha\beta}$ are 8×8 symmetric matrices (a D=10 generalization of the relativistic Pauli matrices) obeying (19).

此处 $\Gamma^{\underline{m}}_{\alpha\beta}$ 是满足式 (19) 的 8×8 对称矩阵 (相对论泡利矩阵的 D=10 推广)

Using the algebra (44) we find that self-consistency conditions for the superem-bedding equation (49), $D_qE_p^m + D_pE_q^m = 0$, imply

利用代数 (44) 我们可得，超嵌入方程 (49) 的自洽性条件 $D_qE_p^m + D_pE_q^m = 0$ 蕴涵

$$D_q\Theta\Gamma^{\underline{m}}D_p\Theta = \delta_{qp}E_0^{\underline{m}}, \tag{51}$$

where

其中

$$E_0^{\underline{m}} = \partial_\xi X^{\underline{m}} - i\partial_\xi \Theta^{\underline{\alpha}} \Gamma^{\underline{m}}_{\underline{\alpha\beta}} \Theta^{\underline{\beta}}.$$

Considering this relation for equal q and p , for instance for $q = p = 1$, we find that, due the to gamma-matrix identities (19), it implies the light-likeness of the ten-vector superfield $E_0^{\underline{m}}$:

当考虑该关系对相等的 q 和 p 成立时，例如对 $q = p = 1$ ，我们发现，由于伽马矩阵恒等式 (19)，它蕴涵十矢量超场 $E_0^{\underline{m}}$ 类光:

$$E_0^{\underline{m}} = D_1 \Theta^{\underline{\alpha}} \Gamma^{\underline{m}}_{\underline{\alpha\beta}} D_1 \Theta^{\underline{\beta}} \;\Rightarrow\; E_{0\underline{m}} E_0^{\underline{m}} = 0. \tag{52}$$

Similarly, the leading component ($\eta^q = 0$ part) of the superfield relation (51)

类似地，超场关系 (51) 的领头分量 ($\eta^q = 0$ 部分

$$\lambda^{\underline{\alpha}}_{\underline{q}} \Gamma^{\underline{m}}_{\underline{\alpha\beta}} \lambda^{\underline{\beta}}_{\underline{p}} = \delta_{qp} \left(\partial_\xi x^{\underline{m}} - i\partial_\xi \theta \Gamma^{\underline{m}} \theta \right) \tag{53}$$

implies the light-likeness of the superparticle momentum

蕴涵超粒子动量是类光的

$$p^{\underline{m}} \propto \left(\partial_\xi x^{\underline{m}} - i\partial_\xi \theta \Gamma^{\underline{m}} \theta \right), \; p^{\underline{m}} p_{\underline{m}} = 0. \tag{54}$$

The relation (53) thus provides a 10D generalization of the four-dimensional Cartan-Penrose (twistor) representation of a light-like vector.

因此关系式 (53) 给出了四维卡坦-彭罗斯 (扭量) 类光矢量表示的 10 维推广。

Moreover, the relations (53) impose strong algebraic constraints on the bosonic spinors λ^{α}_{q} reducing drastically the number of their independent components. Using the Lorentz-harmonic formalism of [33-35] (see [60,68,69] for a recent discussion), one can show that, taking into account certain gauge symmetries of this model, the constrained $\lambda^{\underline{\alpha}}_{q}$ can be identified with homogeneous coordinates of the space $\mathbb{R} \otimes \mathbb{S}^9$, where $\mathbb{S}^9$ is a celestial sphere.

此外，关系式 (53) 对玻色旋量 λ^{α}_{q} 施加了很强的代数约束，大幅减少了其独立分量的数量。利用 [33-35] 的洛伦兹调和形式 (近期讨论见 [60,68,69]) 可以证明，考虑到该模型的 certain 规范对称性后，受约束的 $\lambda^{\underline{\alpha}}_{q}$ 可以等同于空间 $\mathbb{R} \otimes \mathbb{S}^9$ 的齐次坐标，其中 $\mathbb{S}^9$ 是天球面。

It can be shown [84] that the superembedding equation reduces the field content of the coordinate superfields (47) and (48) to the leading components $x^{\underline{m}}(\xi)$ and $\theta^{\underline{\alpha}}(\xi)$ and to the bosonic spinors $\lambda^{\underline{\alpha}}_{q}(\xi)$ constrained by (53). However, it does not produce equations of motion of the superparticle under consideration. For instance, in view of (54), equation (53) would be equivalent to the particle equation of motion $\partial_\xi p^{\underline{m}} = 0$ iff the bosonic spinors $\lambda_q^{\underline{\alpha}}(\xi)$, or at least their bilinear combination in the l.h.s. of (62) were constant. However, neither this nor the equations of motion for the fermionic coordinate functions $\theta^{\underline{\alpha}}(\xi)$ follow from the superembedding condition.

可以证明 [84]，超嵌入方程将坐标超场 (47) 和 (48) 的场内容约化为领头分量 $x^{\underline{m}}(\xi)$ 和 $\theta^{\underline{\alpha}}(\xi)$ ，以及受 (53) 约束的玻色旋量 $\lambda_q^{\underline{\alpha}}(\xi)$ 。但它并未给出所研究超粒子的运动方程。例如，根据 (54)，方程 (53) 等价于粒子运动方程 $\partial_\xi p^{\underline{m}} = 0$ 当且仅当玻色旋量 $\lambda_q^{\underline{\alpha}}(\xi)$ (或至少它们在 (62) 左侧的双线性组合) 是常数。但超嵌入条件既推不出这个结论，也推不出费米坐标函数 $\theta^{\underline{\alpha}}(\xi)$ 的运动方程。

The fact that in the case under consideration, as in the cases considered in sections ”The Worldline Superfield Action for the $N=1, D=3$ Superparticle” and ”The $N=1, D=10$ Superparticle with $n=2$ Worldline Supersymmetry and Pure Spinors,” the superembedding condition does not have dynamical equations among its consequences, implies that there exists a worldline superfield action for the $N=1, D=10$ superparticle with manifest $n=8$ worldline supersymmetry [84] similar to the actions (26) and (39):

正如在“$N=1, D=3$ 超粒子的世界线超场作用量”和“具有 $n=2$ 世界线超对称与纯旋量的 $N=1, D=10$ 超粒子”小节讨论的情形一样，在当前情形下超嵌入条件的推论中不包含动力学方程，这意味着存在具有明显 $n=8$ 世界线超对称的 $N=1, D=10$ 超粒子的世界线超场作用量 [84]，与作用量 (26) 和 (39) 类似:

$$S = \int d\xi d^8\eta P_{\underline{m}}^q \left(D_q X^{\underline{m}} - iD_q \Theta \Gamma^{\underline{m}} \Theta\right). \tag{55}$$

The variation of this action with respect to the Grassmann-odd Lagrange multiplier superfield $P_m^q(\xi, \eta^q)$ produces the superembedding condition (49) as desired. But a nontrivial problem is to prove that this Lagrange multiplier does not contain superfluous dynamical degrees of freedom. To this end it is important to notice that the action (55) is invariant under the following local symmetry transformation:

该作用量对格拉斯曼奇数拉格朗日乘子超场 $P_m^q(\xi, \eta^q)$ 变分恰好得到超嵌入条件 (49)。但一个关键问题是要证明这个拉格朗日乘子不包含多余的动力学自由度。为此需要注意，作用量 (55) 在以下局域对称变换下不变:

$$\delta P_{\underline{m}}^q = D_p \left(\sum\nolimits^{qpr\underline{\alpha}} \Gamma_{\underline{m}\underline{\alpha}\underline{\beta}} D_r \Theta^{\underline{\beta}}\right), \ \delta X^{\underline{m}} = 0, \ \delta\Theta^{\underline{\alpha}} = 0. \tag{56}$$

where the spinor superfield $\sum^{qpr\underline{\alpha}}(\xi, \eta)$ parameter is symmetric and traceless in its $SO(8)$ indices

其中旋量超场 $\sum^{qpr\underline{\alpha}}(\xi, \eta)$ 参数在其 $SO(8)$ 指标下是对称且无迹的

$$\sum\nolimits^{qpr\underline{\alpha}} = \sum\nolimits^{pqr\underline{\alpha}} = \sum\nolimits^{(qpr)\underline{\alpha}}, \ \sum\nolimits^{qqr\underline{\alpha}} = 0. \tag{57}$$

Solving the equations of motion of the coordinate superfields $X^{\underline{m}}(\xi, \eta)$ and $\Theta^{\underline{m}}(\xi, \eta)$

求解坐标超场 $X^{\underline{m}}(\xi, \eta)$ 和 $\Theta^{\underline{m}}(\xi, \eta)$ 的运动方程

$$D_q P_{\underline{m}}^q = 0, \ P_{\underline{m}}^q D_q \Theta \Gamma^{\underline{m}} = 0 \tag{58}$$

and using the above gauge symmetry, one can reduce the Lagrange multiplier superfield to just one term:

并利用上述规范对称性，可将拉格朗日乘子超场约化为仅一项:

$$P^q_{\underline{m}} = \frac{1}{7!}\varepsilon^{qp_1\cdots p_7}\eta^{p_1}\ldots\eta^{p_7}p_{\underline{m}}, \tag{59}$$

where $p_{\underline{m}}$ is a constant vector

其中 $p_{\underline{m}}$ 是常向量

$$\frac{d}{d\xi}p_{\underline{m}} = 0 \tag{60}$$

It is expressed through the bilinear combinations of the bosonic spinors λ^{α}_{q} and hence is light-like:

它由玻色旋量 λ^{α}_{q} 的双线性组合表示，因此是类光的:

$$p_{\underline{m}} \propto \lambda_q\Gamma_{\underline{m}}\lambda_q \;\Rightarrow\; p_{\underline{m}}p^{\underline{m}} = 0. \tag{61}$$

Furthermore, taking into account the consequence (53) of the superembedding condition (49), we conclude that (61) implies

此外，考虑超嵌入条件 (49) 的推论 (53)，我们得到 (61) 蕴含

$$\partial_\xi x^{\underline{m}} - i\partial_\xi\theta\Gamma^{\underline{m}}\theta = ep^{\underline{m}} \tag{62}$$

with some function $e(\xi)$ which can be gauge fixed to a constant using the (super)conformal symmetry of the action (55). This completes the proof of the equivalence of the doubly supersymmetric model described by the action (55) and the Green-Schwarz-like formulation of the $N = 1, D = 10$ massless superparticle mechanics, with the κ -symmetry being related to the $n = 8$ local worldline supersymmetry in a way similar to that discussed in section "Local Worldvolume Supersymmetry Versus κ -Symmetry."

其中存在某个函数 $e(\xi)$，可以利用作用量 (55) 的 (超) 共形对称性通过规范固定将其变为常数。至此我们证明了，由作用量 (55) 描述的双超对称模型等价于格林-施瓦茨形式的 $N = 1, D = 10$ 无质量超粒子力学，其中 κ 对称性与 $n = 8$ 局域世界线超对称的关联方式和小节“局域世界体积超对称与 κ 对称性”中的讨论类似。

Note that in the description of type IIA and type IIB $D = 10$ superparticles, superstrings, and D-p-branes in worldvolume superspace with $n = 16$ super-symmetry (which is the number of independent κ -symmetries in these cases), the superembedding condition puts the theories on the mass shell, i.e., it contains the dynamical equations among its consequences. As a result one cannot use an $n = 16$ superfield generalization of the action (55) to describe type IIA and type IIB super-p-branes. In these cases the superfield equations can be obtained from the generalized action principle [150-152] which is a super-p-brane counterpart of the so-called rheonomic (group manifold) approach to supergravity [153-156].

请注意，在具有 $n=16$ 超对称性 (即上述情形中独立 κ 对称性的数量) 的世界体积超空间中描述 IIA 型和 IIB 型 $D=10$ 超粒子、超弦和 D-p 膜时，超嵌入条件会令理论处于质壳上，也就是说，其推论中包含动力学方程。因此，无法用式 (55) 作用量的 $n=16$ 超场推广来描述 IIA 型和 IIB 型超 p 膜。这类情形下，超场方程可由广义作用量原理 [150-152] 得到，该原理是超引力的所谓流形 (群流形) 方法 [153-156] 对应的超 p 膜版本。

Superembedding Description of Superstrings

超弦的超嵌入描述

$N=1, D=10$ (Heterotic) Superstring

$N=1, D=10$ (杂化) 超弦

To replace all the κ -symmetries of the $N=1, D=10$ superstring with manifest worldsheet supersymmetry, we should introduce an $n=(8,0)$ worldsheet superspace parametrized by coordinates:

为了用明显的世界面超对称替代 $N=1, D=10$ 超弦的所有 κ 对称性，我们应当引入一个由坐标参数化的 $n=(8,0)$ 世界面超空间:

$$z^M = (\xi^m, \eta^q) = (\xi^+, \xi^-, \eta^q), \; q = 1, \dots, 8, \tag{63}$$

where $\xi^\pm$ are light-cone coordinates of the worldsheet which are transformed under the $2d$ Lorentz group $SO(1,1)$ with the Lorentz weights ± 1 , respectively, while η^q are Grassmann-odd $2d$ Majorana-Weyl spinors which are transformed under $SO(1,1)$ with the Lorentz weight $+\frac{1}{2}$, i.e., they are "left-handed."

其中 $\xi^\pm$ 是世界面的光锥坐标，它们分别在洛伦兹权重为 ± 1 的 $2d$ 洛伦兹群 $SO(1,1)$ 下变换，而 η^q 是格拉斯曼奇的 $2d$ 马约拉纳-外尔旋量，它们在 $SO(1,1)$ 下以洛伦兹权重 $+\frac{1}{2}$ 变换，即它们是“左手征”的。

We consider the following action on the superworldsheet coordinates of a superconformal symmetry parametrized by the superfield $\Lambda^+(\xi^+, \eta^q)$ depending on the "left-handed" bosonic and fermionic coordinates only:

我们考虑在超世界面坐标上，由仅依赖“左手征”玻色坐标和费米坐标的超场 $\Lambda^+(\xi^+, \eta^q)$ 参数化的超共形对称的如下作用量:

$$\delta\xi^+ = \Lambda^+ - \frac{1}{2}\eta^q D_q \Lambda^+, \; \delta\eta^q = -\frac{i}{2} D_q \Lambda^+, \; \delta\xi^- = 0. \tag{64}$$

Note that the "right-moving" coordinate ξ^- is inert under these supersymmetry transformations.

注意“右运动”坐标 ξ^- 在这些超对称变换下是不变的。

In (64), D_q are the fermionic covariant derivatives (similar to (43)):

注意在 (64) 中，D_q 是费米协变导数 (类似 (43)):

$$D_q = \partial_q + i\eta^q \partial_+,$$

that obey the $n = (8,0)$ supersymmetry algebra

其满足 $n = (8,0)$ 超对称代数

$$\{D_q, D_p\} = 2i\delta_{qp}\partial_+. \tag{65}$$

A key property of the superconformal symmetry is the homogeneous transformation of the covariant derivatives:

超共形对称的一个关键性质是协变导数的齐次变换:

$$\delta D_q = \frac{i}{2}\left(D_q D_p \Lambda^+\right) D_p.$$

The other object which transforms homogeneously under this superconformal symmetry is the Volkov-Akulov vielbein of the flat $d = 2, n = (8,0)$ superspace:

另一个在该超共形对称下齐次变换的对象是平坦 $d = 2, n = (8,0)$ 超空间的沃尔科夫-阿库洛夫标架:

$$e^+ = d\xi^+ - id\eta^q \eta^q \tag{66}$$

$$\delta e^+ = e^+ \partial_+ \Lambda^+.$$

The superfield parametrizing these superconformal transformations appears as formal contraction of this VA 1-form with the variation symbol:

参数化这些超共形变换的超场表现为该 VA 1-形式与变分符号的正规缩并:

$$i_\delta e^+ \ := \ \delta\xi^+ - i\delta\eta^q \eta^q = \Lambda^+.$$

The coordinate functions are now worldsheet superfields:

坐标函数现在是世界面超场:

$$X^{\underline{m}}(\xi^m, \eta^p) = x^{\underline{m}}(\xi^m) + i\eta^q \chi_q^{\underline{m}}(\xi^m) + \ldots + \frac{1}{8!}\eta^{q_8} \ldots \eta^{q_1} \varepsilon_{q_1 \ldots q_8} y^{\underline{m}}(\xi^m), \tag{67}$$

$$\Theta^{\underline{\alpha}}(\xi^m, \eta^p) = \theta^{\underline{\alpha}}(\xi^m) + \eta^q \lambda_q^{\underline{\alpha}}(\xi^m) + \ldots + \frac{1}{8!}\eta^{q_8} \ldots \eta^{q_1} \varepsilon_{q_1 \ldots q_8} \zeta^{\underline{\alpha}}(\xi^m). \tag{68}$$

The superembedding condition is again a particular case of (13):

超嵌入条件再次是 (13) 的一个特殊情况:

$$E_q^{\underline{m}} = D_q X^{\underline{m}} - i D_q \Theta \Gamma^{\underline{m}} \Theta = 0 \tag{69}$$

which implies that the pullback of the one-form $E^{\underline{m}} = dX^{\underline{m}} - id\Theta\Gamma^{\underline{m}}\Theta$ is

它意味着 1-形式 $E^{\underline{m}} = dX^{\underline{m}} - id\Theta\Gamma^{\underline{m}}\Theta$ 的拉回是

$$E^{\underline{m}} = e^{+} E_{+}^{\underline{m}} + e^{-} E_{=}^{\underline{m}}, \tag{70}$$

where

其中

$$E_{+}^{\underline{m}} = \partial_{+} X^{\underline{m}} - i\partial_{+} \Theta^{\underline{\alpha}} \Gamma_{\underline{\alpha\beta}}^{\underline{m}} \Theta^{\underline{\beta}}, \; E_{=}^{\underline{m}} = \partial_{\_} X^{\underline{m}} - i\partial_{\_} \Theta^{\underline{\alpha}} \Gamma_{\underline{\alpha\beta}}^{\underline{m}} \Theta^{\underline{\beta}}. \tag{71}$$

As in the superparticle case, studying the self-consistency conditions for the superembedding equation (69) using the algebra (65), we find that

和超粒子情况一样，利用代数 (65) 研究超嵌入方程 (69) 的自洽性条件，我们得到

$$D_q \Theta \Gamma^{\underline{m}} D_p \Theta = \delta_{qp} E_{+}^{\underline{m}} \tag{72}$$

which implies

它意味着

$$E_{+\underline{m}} E_{+}^{\underline{m}} = 0. \tag{73}$$

Similarly, the leading component of (72)

类似地，(72) 的主导分量

$$\lambda_q^{\underline{\alpha}} \Gamma_{\underline{\alpha\beta}}^{\underline{m}} \lambda_p^{\underline{\beta}} = \delta_{qp} \left(\partial_{+} x^{\underline{m}} - i\partial_{+} \theta \Gamma^{\underline{m}} \theta\right) \tag{74}$$

implies the left-handed Visrasoro constraint

给出了左行 Virasoro 约束

$$\left(\partial_{+} x^{\underline{m}} - i\partial_{+} \theta \Gamma^{\underline{m}} \theta\right)^2 = 0, \tag{75}$$

the counterpart of the light-likeness of the momentum of the massless superparticle (54).

即 (54) 中无质量超粒子动量类光条件的对应物。

As in the case of the $N = 1, n = 8$ superparticle, the relations (74) impose strong algebraic constraints on the bosonic spinors $\lambda^{\underline{\alpha}}_{q}$ making possible to identify them, up to a scalar multiplier, with Lorentz spinor harmonic variables. Furthermore, similar to the superparticle case, the superembedding condition reduces the dynamical field content of the superfields (67) and (68) to their leading components, $x^{\underline{m}}(\xi^{\pm})$ and $\theta^{\underline{\alpha}}(\xi^{\pm})$, and the bosonic spinors $\lambda^{\underline{\alpha}}_{q}(\xi^{\pm})$ constrained by (74) [86]. The dynamical equations of motion of these fields are not part of the superembedding condition, so one can construct an $n = (8, 0)$ worldsheet superspace action which will produce the superembedding condition and the dynamical equations of motion of the $N = 1$, $D = 10$ superstring.

> 与 $N = 1, n = 8$ 超粒子的情况相同，关系式 (74) 对玻色型旋量 $\lambda^{\underline{\alpha}}_{q}$ 施加了很强的代数约束，使得我们可以在相差一个标量乘子的前提下将其等同于洛伦兹旋量调和变量。此外，和超粒子的情况类似，超嵌入条件将超场 (67) 和 (68) 的动力学场内容约化为其主导分量 $x^{\underline{m}}(\xi^{\pm})$ 和 $\theta^{\underline{\alpha}}(\xi^{\pm})$，以及受 (74) 约束的玻色型旋量 $\lambda^{\underline{\alpha}}_{q}(\xi^{\pm})$ [86]。这些场的动力学运动方程并不包含在超嵌入条件中，因此我们可以构造一个 $n = (8, 0)$ 世界面超空间作用量，它会同时给出超嵌入条件和 $N = 1$ 、$D = 10$ 超弦的动力学运动方程。

Notice, however, that the right-handed Virasoro constraint of the string theory

> 但需要注意，弦理论的右行 Virasoro 约束

$$(\partial\_x^{\underline{m}} - i\partial\_\theta\Gamma^{\underline{m}}\theta)^2 = 0 \tag{76}$$

does not follow from the superembedding condition (69). To obtain the second Virasoro constraint from the superstring action, one should consider a slightly curved geometry on the superworldsheet $\mathcal{M}_{sw}$ which is described by the following worldsheet supervielbein:

> 不能从超嵌入条件 (69) 推出。要从超弦作用量得到第二个 Virasoro 约束，我们需要考虑超世界面 $\mathcal{M}_{sw}$ 上略微弯曲的几何，该几何由下述世界面超标架描述:

$$e^{A} = (e^{+}, e^{-}, e^{q})$$

$$e^{+} = d\xi^{+} - id\eta^{q}\eta^{q},\ e^{q} = d\eta^{q},\ e^{-} = d\xi^{-} + \frac{i}{8}e^{+}D_{q}e^{-}_{q} + e^{q}e^{-}_{q}. \tag{77}$$

It contains the same flat supervielbeins in the left-handed sector and a more complicated right-handed einbein e^{-} whose components are expressed in terms of the superfield $e^{-}_{q}(\xi^{m}, \eta^{p})$ which obeys the constraint:

> 它的左行区和平直超标架相同，而右行区的爱因斯坦贝因 e^{-} 更复杂，其分量可由满足如下约束的超场 $e^{-}_{q}(\xi^{m}, \eta^{p})$ 表示:

$$D_{(p}e^{-}_{q)} = \frac{1}{8}\delta_{qp}D_{r}e^{-}_{r}. \tag{78}$$

The worldvolume component $D_{r}e^{-}_{r}|_{\eta^{q}=0} = e^{--}(\xi^{m})$ is an auxiliary field whose equation of motion obtained from the action (80) considered below will produce the left-handed Virasoro constraint (76).

世界体积分量 $D_r e_r^-|_{\eta^q=0} = e^{--}(\xi^m)$ 是一个辅助场，它的运动方程可以从下文给出的作用量 (80) 得到，该方程会给出左行 Virasoro 约束 (76)。

The corresponding covariant derivatives are

对应的协变导数为

$$\nabla_q = D_q - e_q^- \partial_-, \ \nabla_+ = \partial_+ + \frac{i}{8} D_q e_q^- \partial_-, \ \nabla_- = \partial_-.$$

They form a ”quasi flat” superalgebra with

它们构成了一个“拟平直”超代数，满足

$$\{\nabla_q, \nabla_p\} = 2i\delta_{qp}\nabla_+, \ [\nabla_+, \nabla_q] = 0.$$

For this choice of the worldsheet supergeometry, the superembedding condition takes the form:

对世界面超几何的这种选择，超嵌入条件的形式为:

$$E_q^{\underline{m}} = \nabla_q X^{\underline{m}} - i\nabla_q \Theta \Gamma^{\underline{m}} \Theta = 0. \tag{79}$$

With this superworldsheet geometry at hand, the authors of [86] proposed the following $n = (8,0)\, 2d$ superfield action for the description of $N = 1, D = 10$ superstring:

借助这种超世界面几何，文献 [86] 的作者提出了如下描述 $N = 1, D = 10$ 超弦的 $n = (8,0)\, 2d$ 超场作用量:

$$S = \int d^2\xi d^8\eta P_{\underline{m}}^q \left(\nabla_q X^{\underline{m}} - i\nabla_q \Theta \Gamma^{\underline{m}} \Theta\right) + \int d^2\xi d^8\eta \mathcal{P}^{MN} \left(\mathcal{L}_{MN} - \partial_M Y_N\right).$$

(80)

The first term in this action has the structure similar to the superparticle action (55), the integral of the superembedding condition multiplied by Lagrange multiplier superfield. This term is invariant under local transformations of the Lagrange multiplier similar to those in (56). The second term is new; it is required to generate the string tension and reproduce the Wess-Zumino term of the Green-Schwarz formulation of the superstring. Here $\mathcal{P}^{MN} = -(-)^{MN} \mathcal{P}^{NM}$ is the graded- (anti)symmetric Lagrange multiplier $\left((-)^{MN}\right.$ is a shortcut notation for $(-1)^{\varepsilon(M)\varepsilon(N)}$ where $\varepsilon(M) = \varepsilon\left(z^M\right)$ is the Grassmann parity equal to 0 for the bosons and to 1 for the fermions.), $Y_M(z)$ is an auxiliary superfield, and $\mathcal{L}_{MN}$ are components of the worldsheet superform $\mathcal{L}_2 = \frac{1}{2} dz^N \wedge dz^M \mathcal{L}_{MN}$:

该作用量的第一项结构与超粒子作用量 (55) 类似，是超嵌入条件乘拉格朗日乘子超场的积分。这一项在拉格朗日乘子的局域变换下不变，变换形式和 (56) 中的类似。第二项是新的；它是生成弦张力、重现格林-施瓦茨超弦表述中的韦斯-祖米诺项所必需的。此处 $\mathcal{P}^{MN} = -(-)^{MN}\mathcal{P}^{NM}$ 是分次 (反) 对称拉格朗日乘子 $\left((-)^{MN}\right.$ is a shortcut notation for $(-1)^{\varepsilon(M)\varepsilon(N)}$ (其中 $\varepsilon(M) = \varepsilon\left(z^M\right)$ 是格拉斯曼奇偶性，玻色子为 0，费米子为 1)，$Y_M(z)$ 是辅助超场，$\mathcal{L}_{MN}$ 是世界面超形 $\mathcal{L}_2 = \frac{1}{2}dz^N \wedge dz^M \mathcal{L}_{MN}$ 的分量:

$$\mathcal{L}_2 = B_2 + e^+ \wedge e^- E_+^{\underline{m}} E_{-\underline{m}}, \tag{81}$$

where

其中

$$B_2 = -iE^{\underline{m}} \wedge d\Theta\Gamma_{\underline{m}}\Theta \tag{82}$$

is the pullback of the so-called NS-NS 2-form of flat $N = 1, D = 10$ superspace, and $E_+^{\underline{m}}$ and $E_=^{\underline{m}}$ are components of the decomposition of the pullback of the target superspace vector supervielbein (50) in the covariant basis (77):

是平坦 $N = 1, D = 10$ 超空间中所谓 NS-NS 2-形式的拉回，且 $E_+^{\underline{m}}$ 和 $E_=^{\underline{m}}$ 是目标超空间向量超标架 (50) 的拉回在协变基 (77) 下分解的分量:

$$E^{\underline{m}} = e^+ E_+^{\underline{m}} + e^- E_=^{\underline{m}} + e^q E_q^{\underline{m}}. \tag{83}$$

An important property of the 2-form (81) is that it is closed if the superembedding condition (79) is satisfied:

2-形式 (81) 的一个重要性质是，当超嵌入条件 (79) 满足时，该 2-形式是闭的:

$$d\mathcal{L}_2|_{E_q^{\underline{m}}=0} = 0 \;\rightarrow\; \partial_{[M}\mathcal{L}_{NK)}\big|_{E_q^{\underline{m}}=0} = 0, \tag{84}$$

(where [...) denotes the Grassmann-graded (anti)symmetrization of the indices).

(其中 [...] 表示指标的格拉斯曼分次 (反) 对称化)

This implies that the action is invariant under a gauge symmetry with the superfield parameter:

这说明作用量在以超场为参数的规范对称下不变:

$$\sum^{MNK}(z) = -(-)^{MN}\sum^{NMK}(z) = -(-)^{NK}\sum^{MKN}(z) = \sum^{[MNK)}(z)$$

which acts on the Lagrange multiplier $\mathcal{P}^{MN}$ as follows

该对称对拉格朗日乘子 $\mathcal{P}^{MN}$ 的作用如下

$$\delta \mathcal{P}^{MN} = (-)^{K} \partial_K \sum\nolimits^{MNK} (z)\,. \tag{85}$$

Due to the closer of $\mathcal{L}_{MN}$ modulo the superembedding condition (84), the variation of the action with respect to (85) will be proportional to the superembedding condition and can be therefore compensated by an appropriate transformation of the Lagrange multiplier P_m^{+q} .

由于模超嵌入条件 (84) $\mathcal{L}_{MN}$ 闭，作用量对 (85) 的变分与超嵌入条件成正比，因此可以通过拉格朗日乘子 P_m^{+q} 的适当变换抵消。

The symmetry (85) allows to gauge fix the general solution of the equations of motion for the auxiliary superfield $Y_M(z)$:

对称性 (85) 允许我们规范固定辅助超场 $Y_M(z)$ 运动方程的通解:

$$(-)^{M} \partial_M \mathcal{P}^{MN} = 0$$

to the simple expression

得到如下简单表达式

$$\mathcal{P}^{MN} = \frac{1}{8!} \eta^{q_8} \ldots \eta^{q_1} \varepsilon_{q_1 \ldots q_8} e_{+}^{M} e_{-}^{M} T \tag{86}$$

with a constant T which has the meaning of the superstring tension. Thus the action (80) realizes a dynamical tension generation mechanism for supestrings put forward in [157, 158] (see [66] for more details). It can be shown that the action (80) is classical equivalent to the Green-Schwarz action for an $N = 1, D = 10$ superstring. For a detailed demonstration of this fact, we refer the reader to [86, 108], while here we only note that if we integrate the action (80) over the fermionic variables η^q , impose a conformal gauge on the worldsheet supervielbeins, eliminate all the auxiliary fields by using the local symmetry transformations (56) and (85), and by solving the superembedding condition, then the action (80) reduces to the following one:

其中常数 T 具有超弦张力的物理意义。因此作用量 (80) 实现了 [157, 158] 中提出的超弦动力学张力生成机制 (更多细节见 [66])。可以证明，作用量 (80) 经典等价于 $N = 1, D = 10$ 超弦的格林-施瓦茨作用量。关于这一结论的详细证明我们建议读者参阅文献 [86, 108]，这里仅需指出: 若我们对作用量 (80) 积分掉费米变量 η^q ，对世界面超标架取共形规范，利用定域对称性变换 (56) 和 (85) 消去所有辅助场并求解超嵌入条件，那么作用量 (80) 将约化为:

$$\begin{aligned} T \int \mathcal{L}_2 \Big|_{\eta^q = 0} &= T \int d^2\xi \, (\partial_+ x^{\underline{m}} - i\partial_+ \theta \Gamma^{\underline{m}} \theta) \left(\partial_- x_{\underline{m}} - i\partial_- \theta \Gamma_{\underline{m}} \theta \right) \\ &\quad - iT \int (dx^{\underline{m}} - i d\theta \Gamma^{\underline{m}} \theta) \wedge d\theta \Gamma_{\underline{m}} \theta, \end{aligned} \tag{87}$$

This is the action, in the conformal gauge, for the $N = 1, D = 10$ Green-Schwarz superstring in flat target superspace.

这就是共形规范下，平坦目标超空间中 $N = 1, D = 10$ 格林-施瓦茨超弦的作用量。

To describe within the superembedding approach a fully fledged heterotic string [159] carrying on its worldsheet 32 chiral fermions or 16 chiral bosons generating an $SO(32)$ or $E_8 \times E_8$ gauge group, one should add to the action (80) terms which describe the dynamics of these heterotic degrees of freedom. The construction of such terms with the use of $n = 8$ worldsheet superfields turned out to be not a straightforward enterprise. For different ways of realizing the chiral fermion construction, we refer the reader to [160-162].

要在超嵌入框架下描述完整的杂化弦 [159]——其世界面存在 32 个手征费米子或 16 个手征玻色子，生成 $SO(32)$ 或 $E_8 \times E_8$ 规范群——我们需要在作用量 (80) 中增加描述这些杂化自由度动力学的项。利用 $n = 8$ 世界面超场构造这些项并非易事。关于手征费米子构造的不同方式，我们建议读者参阅文献 [160-162]。

$N = 2, D = 10$ Superstrings with $n = (8, 8)$ Worldsheet Supersymmetry: Equations of Motion from the Superembedding Condition

$N = 2, D = 10$ 超弦，具有 $n = (8, 8)$ 世界面超对称性: 来自超嵌入条件的运动方程

Let us now consider examples of superstrings whose superembedding description produces all the equations of motion, i.e., is entirely on-shell formulation. This is the case of type IIA and type IIB $D = 10$ superstrings.

现在我们来考虑这类超弦的例子: 它们的超嵌入描述可以给出全部运动方程，也就是完全的在壳表述。IIA 型和 IIB 型 $D = 10$ 超弦就属于这种情况。

The $N = 2, D = 10$ target superspace of the type IIA superstring has 32 fermionic directions parametrized by 2 Majorana-Weyl spinor coordinates of opposite chiralities:

IIA 型超弦的 $N = 2, D = 10$ 目标超空间有 32 个费米子方向，由 2 个手征性相反的马约拉纳-外旋量坐标参数化:

$$IIA: \ Z^{\underline{M}} = \left(X^{\underline{m}}, \Theta^{\underline{\alpha}1}, \Theta^{2}_{\underline{\alpha}}\right), \ \underline{\alpha} = 1, \dots, 16 \tag{88}$$

and in the type IIB superstring case, 32 fermionic directions of the target superspace are parametrized by 2 Majorana-Weyl spinor coordinates of the same chirality:

而在 IIB 型超弦的情况中，目标超空间的 32 个费米子方向由 2 个手征性相同的马约拉纳-外旋量坐标参数化:

$$IIB: \ Z^{\underline{M}} = \left(X^{\underline{m}}, \Theta^{\underline{\alpha}1}, \Theta^{\underline{\alpha}2}\right), \ \underline{\alpha} = 1, \dots, 16. \tag{89}$$

Since the superembedding descriptions of the two cases are quite similar, here we will only discuss the type IIB case, because, as we will show below, in this case the superembedding approach provides us with a universal description of the fundamental type IIB superstring and a Dirichlet string also called the super-D1-brane [89, 163].

由于两种情况的超嵌入描述十分相似，我们在此仅讨论 IIB 型的情况，因为我们下文会说明，在这种情况下超嵌入方法可以为基础 IIB 型超弦和狄利克雷弦 (也称为超 D1 膜 [89, 163]) 提供统一描述。

To replace all the 16 kappa-symmetries of the GS formulation of the type IIB superstring with the manifest worldsheet supersymmetry, we shall consider an embedding into the type IIB target superspace of an $n = (8, 8)$ worldsheet superspace $\mathcal{M}_{sw}$ parametrized by coordinates $z^M = \left(\xi^+, \xi^-, \eta^q, \eta^{\dot{q}}\right)$. The eight fermionic coordinates η^q $(q = 1, \ldots, 8)$ are "left-handed," they are Grassmann-odd $2d$ Majorana-Weyl spinors which are transformed under $SO(1, 1)$ with the Lorentz weight $+\frac{1}{2}$. The eight fermionic coordinates $\eta^{\dot{q}}$ are "right-handed"; they are Grassmann- odd $2d$ Majorana-Weyl spinors which are transformed under $SO(1, 1)$ with the Lorentz weight $-\frac{1}{2} \cdot \eta^q$ and $\eta^{\dot{q}}$ transform under different eight-dimensional (s- and c-spinor) representations of $SO(8)$ related to each other by triality.

为了用显式世界面超对称性替代 IIB 型超弦 GS 表述的全部 16 个 κ 对称性，我们考虑将一个 $n = (8, 8)$ 世界面超空间 $\mathcal{M}_{sw}$ 嵌入到 IIB 型目标超空间中，该世界面超空间由坐标 $z^M = \left(\xi^+, \xi^-, \eta^q, \eta^{\dot{q}}\right)$ 参数化。8 个费米坐标 η^q $(q = 1, \ldots, 8)$ 是「左手性」的，它们是格拉斯曼奇的 $2d$ 马约拉纳-外旋量，在 $SO(1, 1)$ 下变换，洛伦兹权为 $+\frac{1}{2}$。8 个费米坐标 $\eta^{\dot{q}}$ 是「右手性」的，它们是格拉斯曼奇的 $2d$ 马约拉纳-外旋量，在 $SO(1, 1)$ 下变换，洛伦兹权为 $-\frac{1}{2} \cdot \eta^q$，且 $\eta^{\dot{q}}$ 按三一性关联的 $SO(8)$ 的两个不同八维表示 (s 旋量和 c 旋量) 变换。

For simplicity we will consider the case of the superembedding of the flat superworldsheet. This will result in the on-shell description of the type IIB superstring in the conformal gauge. To get from the superembedding the type IIB superstring equations of motion which are not subject to the conformal gauge, one should deal with a geometry of a curved $n = (8, 8)$ superworldsheet (see [89] and [150, 163, 164] for more details in a bit different setup).

为简化起见，我们考虑平坦超世界面的超嵌入情况，这将得到共形规范下 IIB 型超弦的在壳描述。若要从超嵌入得到不受共形规范约束的 IIB 型超弦运动方程，需要处理弯曲 $n = (8, 8)$ 超世界面的几何 (关于稍有不同的设定的更多细节见 [89] 和 [150, 163, 164])。

The supersymmetry invariant supervielbeins of the flat $n = (8, 8)$ superworld-sheet are

平坦 $n = (8, 8)$ 超世界面的超对称不变 supervielbein(超标架场) 为

$$e^+ = d\xi^+ - id\eta^q\eta^q,\ e^q = d\eta^q,$$

$$e^- = d\xi^- - id\eta^{\dot{q}}\eta^{\dot{q}},\ e^{\dot{q}} = d\eta^{\dot{q}}. \tag{90}$$

The corresponding fermionic covariant derivatives are

对应的费米协变导数是

$$D_q = \partial_q + i\eta^q\partial_+,\ D_{\dot{q}} = \partial_{\dot{q}} + i\eta^{\dot{q}}\partial_-. \tag{91}$$

They obey the rigid $n=(8,8)$ supersymmetry algebra:

它们满足刚性 $n=(8,8)$ 超对称代数:

$$\left\{D_q, D_p\right\} = 2i\delta_{qp}\partial_{+},\ \left\{D_q, D_{\dot{p}}\right\} = 0,\ \left\{D_{\dot{q}}, D_{\dot{p}}\right\} = 2i\delta_{\dot{q}\dot{p}}\partial_{-}. \tag{92}$$

We require that the $n=(8,8)$ superworldsheet is embedded into a flat type IIB $D=10$ superspace (89) in such a way that the pullback of the Volkov-Akulov one-form

我们要求 $n=(8,8)$ 超世界面嵌入到平坦 IIB 型 $D=10$ 超空间 (89) 中，满足沃尔科夫-阿库洛夫一元形式的拉回

$$E^{\underline{a}} = dZ^{\underline{M}} E_{\underline{M}}^{\underline{a}} = dX^{\underline{a}} - id\Theta^1\Gamma^{\underline{a}}\Theta^1 - id\Theta^2\Gamma^{\underline{a}}\Theta^2 \tag{93}$$

on the tangent space basis (90) of the superworldsheet is nonzero only along the bosonic directions

在超世界面切空间基底 (90) 上仅沿玻色方向非零

$$\begin{aligned} E^{\underline{m}} &= e^{+}E_{+}^{\underline{m}} + e^{-}E_{-}^{\underline{m}} \\ &= e^{+}\left(\partial_{+}X^{\underline{m}} - \partial_{+}\Theta^1\Gamma^{\underline{m}}\Theta^1 - i\partial_{+}\Theta^2\Gamma^{\underline{m}}\Theta^2\right) \\ &+ e^{-}\left(\partial_{-}X^{\underline{m}} - \partial_{-}\Theta^1\Gamma^{\underline{m}}\Theta^1 - i\partial_{-}\Theta^2\Gamma^{\underline{m}}\Theta^2\right). \end{aligned} \tag{94}$$

This implies the conventional form of the superembedding conditions:

这给出了常规形式的超嵌入条件:

$$\begin{aligned} E_{q}^{\underline{m}} &= D_q X^{\underline{m}} - D_q\Theta^1\Gamma^{\underline{m}}\Theta^1 - iD_q\Theta^2\Gamma^{\underline{m}}\Theta^2 = 0, \\ E_{\dot{q}}^{\underline{m}} &= D_{\dot{q}} X^{\underline{m}} - D_{\dot{q}}\Theta^1\Gamma^{\underline{m}}\Theta^1 - iD_{\dot{q}}\Theta^2\Gamma^{\underline{m}}\Theta^2 = 0. \end{aligned} \tag{95}$$

The self-consistency conditions which follow from (95) upon applying to the latter the covariant derivatives D_p and $D_{\dot{p}}$ and symmetrizing the $SO(8)$ indices are

对 (95) 应用协变导数 D_p 和 $D_{\dot{p}}$ 并对 $SO(8)$ 指标对称化后得到的自洽条件为

$$D_q E_p^m + D_p E_q^m = 0,\ D_{\dot{q}} E_{\dot{p}}^m + D_{\dot{p}} E_{\dot{q}}^m = 0, \tag{96}$$

and

以及

$$D_q E_{\dot{p}}^{\underline{m}} + D_{\dot{p}} E_q^{\underline{m}} = 0. \tag{97}$$

In view of the anti-commutation relations (92), these conditions take the following form:

利用对易关系 (92)，这些条件可写为如下形式:

$$\delta_{qp} E_{+}^{\underline{m}} = D_q \Theta^1 \Gamma^{\underline{m}} D_p \Theta^1 + D_q \Theta^2 \Gamma^{\underline{m}} D_p \Theta^2, \tag{98}$$

$$\delta_{\dot{q}\dot{p}} E_{-}^{\underline{m}} = D_{\dot{q}} \Theta^1 \Gamma^{\underline{m}} D_{\dot{p}} \Theta^1 + D_{\dot{q}} \Theta^2 \Gamma^{\underline{m}} D_{\dot{p}} \Theta^2, \tag{99}$$

and

和

$$D_q \Theta^1 \Gamma^{\underline{m}} D_{\dot{p}} \Theta^1 + D_q \Theta^2 \Gamma^{\underline{m}} D_{\dot{p}} \Theta^2 = 0. \tag{100}$$

Fundamental String Solution of the Superembedding Conditions

超嵌入条件的基本弦解

The simplest particular solution of equation (100) is

方程 (100) 最简单的特解为

$$D_{\dot{p}} \Theta^{1\underline{\alpha}} = 0, \; D_p \Theta^{2\underline{\alpha}} = 0 \tag{101}$$

which, in view of the superalgebra (92), implies

结合超代数 (92)，上式可推导出

$$\partial_{-} \Theta^{1\underline{\alpha}} = 0, \; \partial_{+} \Theta^{2\underline{\alpha}} = 0. \tag{102}$$

So the leading components of the fermionic superfields $\Theta^{1,2}|_{\eta=0} = \theta^{1,2}(\xi^m)$ obey the same equations of motion as the fermionic coordinate functions of the type IIB Green-Schwarz superstring formulation in the conformal gauge.

因此，费米子超场 $\Theta^{1,2}|_{\eta=0} = \theta^{1,2}(\xi^m)$ 的领头分量满足的运动方程，与共形规范下 IIB 型格林-施瓦茨超弦 formulation 的费米子坐标函数满足的运动方程一致。

Then (98) and (99) reduce to

此时 (98) 和 (99) 可简化为

$$\delta_{qp}E_{+}^{\underline{m}} = D_q\Theta^1\Gamma^{\underline{m}}D_p\Theta^1, \tag{103}$$

$$\delta_{\dot{q}\dot{p}}E_{=}^{\underline{m}} = D_{\dot{q}}\Theta^2\Gamma^{\underline{m}}D_{\dot{p}}\Theta^2, \tag{104}$$

which (as a consequence of the Fierz identities (19)) imply the left- and right-handed Virasoro constraints

作为费尔兹恒等式 (19) 的推论，上式给出左手和右手维拉索罗约束

$$E_{+}^{\underline{m}}E_{+\underline{m}} = 0,\ E_{=}^{\underline{m}}E_{-\underline{m}} = 0, \tag{105}$$

where, in view of (102), E_{+}^{m} and E_{-}^{m} reduce to

其中，结合 (102)，E_{+}^{m} 和 E_{-}^{m} 可简化为

$$E_{+}^{\underline{m}} = \partial_{+}X^{\underline{m}} - i\partial_{+}\Theta^{\alpha 1}\Gamma^{\underline{m}}_{\underline{\alpha\beta}}\Theta^{\beta 1}_{-},\ E_{=}^{\underline{m}} = \partial_{-}X^{\underline{m}} - i\partial_{-}\Theta^{\alpha 2}\Gamma^{\underline{m}}_{\underline{\alpha\beta}}\Theta^{\beta 2}_{-}. \tag{106}$$

Acting on Eq. (103) with ∂_{-} and on (104) with ∂_{+} , and using (102), we get the equation of motion of the bosonic vector superfield $X^{\underline{m}}$:

用 ∂_{-} 作用于方程 (103)，用 ∂_{+} 作用于方程 (104)，再利用 (102)，我们得到玻色矢量超场 $X^{\underline{m}}$ 的运动方程:

$$\partial_{-}\partial_{+}X^{\underline{m}}(\xi,\eta^q,\eta^{\dot{q}}) = 0\ \Rightarrow\ \partial_{-}\partial_{+}x^{\underline{m}}(\xi) = 0. \tag{107}$$

Its leading component coincides with the bosonic field equation of the type IIB superstring in the conformal gauge.

其领头分量与共形规范下 IIB 型超弦的玻色场方程一致。

D1-Brane Solution of the Superembedding Condition

D1 膜的超嵌入条件解

As was observed in [89], Eq. (101) does not describe the general solution of Eq. (100). The general solution is an $O(2)$ rotated version of (101):

正如文献 [89] 中指出，式 (101) 并未描述式 (100) 的通解。通解是式 (101) 经 $O(2)$ 旋转得到的形式:

$$\cos\phi D_{\dot{p}}\Theta^{1\underline{\alpha}} - \sin\phi D_{\dot{p}}\Theta^{2\underline{\alpha}} = 0,\ \sin\phi D_p\Theta^{1\underline{\alpha}} + \cos\phi D_p\Theta^{2\underline{\alpha}} = 0, \tag{108}$$

where, a priori, ϕ is a worldsheet superfield. The study of the self-consistency conditions of this solution carried out in [89] showed that ϕ is actually a constant angle. In [163] the angle ϕ was related to the on-shell constant value of the field strength $F_{mn} = \partial_m A_n - \partial_n A_m$ of a $2d$ Born-Infeld gauge field living on the worldsheet of a D1-brane, namely,

其中，ϕ 先验地是一个世界面超场。文献 [89] 对该解自洽性的研究表明，ϕ 实际上是一个恒定角度。文献 [163] 将角度 ϕ 与 D1 膜世界面上的 $2d$ Born-Infeld 规范场的场强 $F_{mn} = \partial_m A_n - \partial_n A_m$ 的在壳恒定值联系起来，即:

$$\tan\phi = \sqrt{\frac{1 - F^{(0)}}{1 + F^{(0)}}},\ F_0 = \frac{1}{2}\varepsilon^{mn}F_{mn}.$$

Therefore, the solution (108) of the superembedding conditions (95) provides us with the description of the on-shell dynamics of a super-D1-brane of type IIB string theory (see [163] for more details).

因此，超嵌入条件 (95) 的解 (108) 给出了 IIB 型弦理论中超 D1 膜的在壳动力学描述 (更多细节参见文献 [163])。

To summarize, the geometrical condition on the embedding of an $n = (8, 8)$, $d = 2$ supersurface into a type IIA or IIB $D = 10$ superspace provides us with the full set of constraints and dynamical equations of motion for the on-shell description of type II superstrings and D1-branes. This is also the case for the Dp-branes in type II $D = 10$ supergravities and the M2- and M5-branes in $D = 11$ supergravity. It is in this way that the equations of motion of the Dp-branes and the M5-brane were first obtained in [90] and [91] earlier than the actions for these objects were constructed in [113, 114, 141 − 143, 151] .

综上，将 $n = (8, 8)$、$d = 2$ 超曲面嵌入 IIA 或 IIB 型 $D = 10$ 超空间的几何条件，给出了描述 II 型超弦与 D1 膜在壳性质的全套约束条件与动力学运动方程。这一结论同样适用于 II 型 $D = 10$ 超引力中的 Dp 膜，以及 $D = 11$ 超引力中的 M2 膜与 M5 膜。Dp 膜和 M5 膜的运动方程正是通过这种方式，在这些对象的作用量于 [113, 114, 141 − 143, 151] 中构造出来之前，就于文献 [90] 和 [91] 中首次得到了。

Let us note that in the cases in which the superembedding condition puts the theory on the mass shell, it is not possible to construct worldvolume superfield actions similar to those which we presented for $N = 1$ superparticles and superstrings. Instead, in these cases one can use a generalized action principle [150-152] which is a super-p-brane counterpart of the so-called rheonomic (group manifold) approach to supergravity [153-156].

需要注意，当超嵌入条件将理论约束在质量壳时，我们无法构造出类似于我们已经给出的 $N = 1$ 超粒子与超弦的世界体积超场作用量。相反，在这类情况中我们可以使用广义作用量原理 [150-152]，它是超 p 膜对应于超引力的所谓流变 (群流形) 方法的对应形式 [153-156]。

Superembedding Description of M2- and M5-Branes

M2 膜与 M5 膜的超嵌入描述

Let us now consider how the universal superembedding condition (11) produces the full set of the equations of motion of the M-theory membrane [89, 150] and the five-brane [91, 92] .

现在我们来讨论通用超嵌入条件 (11) 如何导出 M 理论膜 [89, 150] 与五膜 [91, 92] 的全套运动方程。

Now the target superspace is that of $D = 11$ supergravity parametrized by the supercoordinates $Z^{\underline{M}} = \left(X^{\underline{m}}, \Theta^{\underline{\mu}}\right)$, where $X^{\underline{m}}\left(\underline{m} = 0, 1, \ldots, 10\right)$ are 11 bosonic space-time coordinates and $\Theta^{\underline{\mu}}\left(\mu = 1, \ldots, 32\right)$ are 32 fermionic (Majorana spinor) coordinates. The geometry of this superspace is described by bosonic and fermionic supervielbeins:

此时的目标超空间是 $D = 11$ 超引力的超空间，由超坐标 $Z^{\underline{M}} = \left(X^{\underline{m}}, \Theta^{\underline{\mu}}\right)$ 参数化，其中 $X^{\underline{m}}\left(\underline{m} = 0, 1, \ldots, 10\right)$ 是 11 个玻色型时空坐标，$\Theta^{\underline{\mu}}\left(\mu = 1, \ldots, 32\right)$ 是 32 个费米型 (马约拉纳旋量) 坐标。该超空间的几何由玻色型和费米型超标架描述:

$$E^{\underline{a}}(Z) = dZ^{\underline{M}} E_{\underline{M}}^{\underline{a}}(Z), \; E^{\underline{\alpha}}(Z) = dZ^{\underline{M}} E_{\underline{M}}^{\underline{\alpha}}(Z), \tag{109}$$

and a spin connection one-form $\Omega^{\underline{ab}}(Z) = dZ^{\underline{M}} \Omega_{\underline{M}}^{\underline{ab}}(Z)$, which is antisymmetric in the indices $\underline{a}$ and $\underline{b}$ and takes values in the vector representation of the local tangent-space (structure) Lorentz group $SO(1, 10)$.

以及自旋联络一元形式 $\Omega^{\underline{ab}}(Z) = dZ^{\underline{M}} \Omega_{\underline{M}}^{\underline{ab}}(Z)$，它在指标 $\underline{a}$ 和 $\underline{b}$ 下反对称，属于局部切空间 (结构) 洛伦兹群 $SO(1, 10)$ 的矢量表示。

The geometry of the $D = 11$ superspace is assumed to satisfy the 11-dimensional supergravity constraints [165, 166], one of the essential ones being the constraint on the bosonic torsion 2-form of the target superspace (in what follows we will not explicitly right the wedge product $\wedge$ of differential forms) (In our conventions the external differential acts on the wedge-product of the differential forms from the right, e.g., in the case of the wedge product of an m-form with an n-form $d(A_m \wedge \; B_n) = A_m \wedge dB_n + (-1)^n dA_m \wedge B_n$. Let us also recall that for the wedge product of two superforms $A_m \wedge B_n = (-1)^{mn} (-1)^{[A][B]} B_n \wedge A_m$, where $[A]$ and $[B]$ stand, respectively, for the Grassmann parity of the forms A_m and B_n.)

我们假设 $D = 11$ 超空间的几何满足 11 维超引力约束 [165, 166]，其中核心约束之一是对目标超空间玻色型挠率二元形式的约束 (下文中我们不会明确写出微分形式的外积 $\wedge$)(按我们的约定，外微分作用在微分形式外积的右侧，例如 m 形式与 n 形式 $d(A_m \wedge \; B_n) = A_m \wedge dB_n + (-1)^n dA_m \wedge B_n$ 的外积就是如此。另外我们也回顾一下，两个超形式 $A_m \wedge B_n = (-1)^{mn} (-1)^{[A][B]} B_n \wedge A_m$ 的外积满足 $[A]$ 和 $[B]$ 分别是形式 A_m 和 B_n 的格拉斯曼奇偶性。)

$$T^{\underline{a}} = dE^{\underline{a}} - E^{\underline{b}} \Omega_{\underline{b}}{}^{\underline{a}} = -i E^{\underline{\alpha}} \Gamma_{\underline{\alpha\beta}}^{\underline{a}} E^{\underline{\beta}}, \tag{110}$$

where $\Gamma_{\underline{\alpha\beta}}^{\underline{a}}$ are the $D = 11$ Dirac matrices in the Majorana representation (see Appendices "Appendix A: $SO(1, 2) \times SO(8)$ Invariant Representation for $D = 11$ Dirac Matrices and M2-Brane Lorentz Harmonics" and "Appendix B: $SO(1, 5) \times SO(5)$ Invariant Representation for $D = 11$ Dirac Matrices and M5-Brane Lorentz Harmonics").

其中 $\Gamma_{\underline{\alpha\beta}}^{\underline{a}}$ 是马约拉纳表示下的 $D = 11$ 狄拉克矩阵 (参见附录“附录 A: $SO(1, 2) \times SO(8)$ 关于 $D = 11$ 狄拉克矩阵与 M2 膜洛伦兹调和的不变表示”和“附录 B: $SO(1, 5) \times SO(5)$ 关于 $D = 11$ 狄拉克矩阵与 M5 膜洛伦兹调和的不变表示”)。

As is well known, one of the reasons to impose the constraint (110) is that in flat limit, it describes the geometry of flat superspace with torsion. In flat superspace the spin connection is flat, i.e., its curvature

$R = d\Omega + \Omega \wedge \Omega$ is zero, so the connection is a pure gauge with respect to local transformations of the structure group $SO(1,10)$. Namely,

众所周知，施加约束 (110) 的原因之一是: 在平坦极限下，该约束描述带挠率的平坦超空间几何。平坦超空间中自旋联络是平坦的，即其曲率 $R = d\Omega + \Omega \wedge \Omega$ 为零，因此对结构群 $SO(1,10)$ 的局域变换而言，该联络是纯规范，即:

$$\Omega_{\underline{a}}{}^{\underline{b}}|_{flat} = u^{-1}{}_{\underline{a}}^{\underline{c}} du_{\underline{c}}{}^{\underline{b}} \tag{111}$$

is the $SO(1,10)$ group Cartan form, where $u_a{}^b(Z)$ are $SO(1,10)$ matrices (whose counterparts on $\mathcal{M}_{sw}$, called Lorentz harmonics, will play an important role in our superembedding description):

是 $SO(1,10)$ 群的嘉当形式，其中 $u_a{}^b(Z)$ 是 $SO(1,10)$ 矩阵 (它们在 $\mathcal{M}_{sw}$ 上的对应物称为洛伦兹调和，将在我们的超嵌入描述中发挥重要作用):

$$u_{\underline{a}}{}^{\underline{c}} u_{\underline{a}}{}^{\underline{d}} \eta_{\underline{cd}} = \eta_{\underline{ab}},\ u_{\underline{a}}{}^{\underline{c}} u_{\underline{a}}{}^{\underline{d}} \eta^{\underline{ab}} = \eta^{\underline{cd}},\ u^{-1} = u^T, \tag{112}$$

and η_{ab} is the $D = 11$ Minkowski metric whose signature is chosen to be ”almost minus.”

且 η_{ab} 是 $D = 11$ 闵氏度规，其符号取为“近似负号”。

Thus, by an appropriate local Lorentz transformation which acts on the vector and spinor components of the supervielbein (109), in flat superspace, one can set $\Omega_{\underline{a}}{}^{\underline{b}} = 0$. In this basis the vector supervielbein of the flat superspace is the Volkov-Akulov one-form (12):

因此，通过作用于 (109) 超 vielbein 的矢量和旋量分量的适当局域洛伦兹变换，我们可以在平坦超空间中令 $\Omega_{\underline{a}}{}^{\underline{b}} = 0$ 。在此基底下，平坦超空间的矢量超 vielbein 就是沃尔科夫-阿库洛夫一元形式 (12):

$$E^{\underline{a}} = dX^{\underline{a}} - id\Theta^{\underline{\alpha}} \Gamma^{\underline{a}}_{\underline{\alpha\beta}} \Theta^{\underline{\beta}}, \tag{113}$$

the spinor supervielbein is the exact 1-form $E^{\underline{\alpha}} = d\Theta^{\underline{\alpha}}$ and the torsion (110) reduces to

旋量超 vielbein 是正合一元形式 $E^{\underline{\alpha}} = d\Theta^{\underline{\alpha}}$ ，而挠率 (110) 约化为

$$T^{\underline{a}}_{\text{flat}} = -id\Theta^{\underline{\alpha}} \Gamma^{\underline{a}}_{\underline{\alpha\beta}} d\Theta^{\underline{\beta}}. \tag{114}$$

Though the superembedding approach has been developed in full generality for the description of branes propagating in curved target superspaces of supergravity theories (see, e.g., [108, 109] for a detailed review), for simplicity in what follows, we will restrict our consideration to the flat $D = 11$ target superspace.

尽管超嵌入方法已被完整推广，用于描述在超引力理论的弯曲目标超空间中传播的膜 (例如，详细综述见 [108, 109])，为简单起见，在下文中我们将讨论限定在平坦 $D = 11$ 目标超空间内。

Worldvolume superspaces of the M2- and M5-brane are parametrized by $d = p + 1$ bosonic coordinates ξ^m (where $p = 2$ for M2 and $p = 5$ for M5) and have 16 fermionic directions associated with maximal $n = 16$ supersymmetries of the $(p+1)$ -dimensional superworldvolume geometry. The number of worldvolume supersymmetries is taken to be the same as the number of κ -symmetries in the Green-Schwarz-like formulations of these branes [6, 113, 114]. The corresponding fermionic coordinates $\eta^{\mu q}$ carry two indices, the index μ which in the case of flat superspace (or after fixing a Wess-Zumino gauge in supergravity superspace) is associated with the index α of a fundamental representation of Spin $(1, p)$ and the index q which labels a fundamental representation of Spin $(10 - p)$. Note that the group Spin $(1, p) \times$ Spin $(10 - p)$ is the subgroup of the $D = 11$ tangent space symmetry group Spin $(1, 10)$ which remains unbroken when the M2- or M5-brane worldvolume is embedded into $D = 11$ target superspace. In summary, the M2- and M5-brane superworldvolume coordinates are

M2 膜和 M5 膜的世界体积超空间由 $d = p + 1$ 个玻色坐标 ξ^m 参数化 (对 M2 膜有 $p = 2$ ，对 M5 膜有 $p = 5$)，并拥有 16 个费米方向，对应于 $(p+1)$ 维超世界体积几何的最大 $n = 16$ 超对称。世界体积超对称的数目与这些膜的格林-施瓦茨类表述中 κ 对称的数目一致 [6, 113, 114]。相应的费米坐标 $\eta^{\mu q}$ 带有两个指标: 指标 μ 在平坦超空间情形 (或在超引力超空间中固定了韦斯-祖米诺规范后) 对应 Spin $(1, p)$ 基本表示的指标 α ，指标 q 则标记 Spin $(10 - p)$ 的一个基本表示。注意群 Spin $(1, p) \times$ Spin $(10 - p)$ 是 $D = 11$ 切空间对称群 Spin $(1, 10)$ 的子群，当 M2 膜或 M5 膜的世界体积嵌入 $D = 11$ 目标超空间时，该子群保持不破缺。综上，M2 膜和 M5 膜的超世界体积坐标为

$$z^M = (\xi^m, \eta^{\mu q}),$$

and the superworldvolume geometry is described by the supervielbeins:

超世界体积几何由以下超 vielbein 描述:

$$e^A = dz^M e_M{}^A(z) = (e^a, e^{\alpha q}), \ p = 2, 5, \begin{cases} m, a = 0, 1, \ldots, p, \\ \mu, \alpha = 1, \ldots, 2^{\left[\frac{p}{2}\right]}, \\ q = 1, \ldots, 2^{\left[\frac{10-p}{2}\right]}, \end{cases} \tag{115}$$

where $\left[\frac{n}{2}\right]$ stands for the integer part of $\frac{n}{2}$.

其中 $\left[\frac{n}{2}\right]$ 代表 $\frac{n}{2}$ 的整数部分。

The superworldvolume pullback of the target-space supervielbeins (109) in the basis of (115) is

目标空间超 vielbein (109) 在基 (115) 下的超世界体积拉回是

$$E^{\underline{a}}(Z(z)) = e^a E_a^{\underline{a}} + e^{\alpha q} E_{\alpha q}^{\underline{a}}, \ E^{\underline{\alpha}}(Z(z)) = e^a E_a^{\underline{\alpha}} + e^{\alpha q} E_{\alpha q}^{\underline{\alpha}}, \tag{116}$$

where

其中

$$E_{\mathcal{A}}^{\underline{A}} = \nabla_{\mathcal{A}} Z^{\underline{M}} E_{\underline{M}}^{\underline{A}}, \ \nabla_{\mathcal{A}} \ := \ e_{\mathcal{A}}^{M}(z)\,\partial_M \tag{117}$$

and $e_{\mathcal{A}}^{M}(z)$ are the components of the matrix inverse to the matrix $e_{M}^{\mathcal{A}}$ of the worldvolume supervielbein components (115).

且 $e_{\mathcal{A}}^{M}(z)$ 是世界体积超 vielbein 分量 (115) 的矩阵 $e_{M}^{\mathcal{A}}$ 的逆矩阵分量。

The superembedding condition is the vanishing of the component of the pullback of $E^{\underline{a}}$ in (116) along the worldvolue fermionic directions $e^{\alpha q}$, i.e.,

超嵌入条件是指 (116) 中 $E^{\underline{a}}$ 拉回沿世界体积费米方向 $e^{\alpha q}$ 的分量为零，即

$$E_{\alpha q}{}^{\underline{a}} = \nabla_{\alpha q} Z^{\underline{M}} E_{\underline{M}}{}^{\underline{a}}(Z(z)) = 0. \tag{118}$$

In the case of superparticles and superstrings, the geometry of the worldline superspace and the worldsheet superspace is superconformally flat which allowed us to consider the superembedding equation with flat fermionic covariant derivatives $\nabla_{\alpha q} = D_{\alpha q}$; see (43) and (91). This is not the case for the higher-dimensional super-p-branes, in particular for the M2-brane and the M5-brane. So, in these cases the construction is invariant under the full group of worldvolume superdiffeomorphisms $z^M \to z'^M(z)$.

对于超粒子和超弦而言，世界线超空间与世界面超空间的几何是超共形平坦的，这使得我们可以讨论带有平坦费米子协变导数 $\nabla_{\alpha q} = D_{\alpha q}$ 的超嵌入方程；参见式 (43) 和 (91)。对于更高维的超 p 膜，尤其是 M2 膜和 M5 膜，情况并非如此。因此，在这些情况中，该构造在整体世界体积超微分同胚群 $z^M \to z'^M(z)$ 下保持不变。

There are to ways to proceed in these cases. The first is to assume some constraints (similar to (110)) on the worldsheet superspace supergravity described by the supervielbeins (115). The second one is to induce the worldvolume super-space supergravity by a specific embedding of the superworldvolume into the target superspace. We will proceed along this way. It requires the introduction of auxiliary superworldvolume variables which have the meaning of $SO(1,10)$ Lorentz harmonics, or so-called (spinor) moving frame variables.

在这些情形下有两种处理思路。第一种是对由超标架 (115) 描述的世界面超空间超引力施加若干约束 (类似式 (110))。第二种是通过将超世界体积特定嵌入到目标超空间中，来诱导出世界体积超空间超引力。我们将采用这种思路，它需要引入辅助超世界体积变量，这些变量具有 $SO(1,10)$ 洛伦兹调和的含义，也就是所谓的 (旋量) 活动标架变量。

Superembedding Condition and the Induced Geometry of the Superworldvolume

超嵌入条件与超世界体积的诱导几何

The superembedding condition (118) can be obtained by specifying the geometry of the superworldvolume $\mathcal{M}_{sw}$ induced by its embedding into the target superspace as follows. We require that, as in the classical

”bosonic” surface theory, the embedding is such that at each point z^M in $\mathcal{M}_{sw}$, the pullback of the target space supervielbein

超嵌入条件 (118) 可通过如下方式确定嵌入目标超空间后诱导得到的超世界体积 $\mathcal{M}_{sw}$ 的几何来获得。我们要求，和经典“玻色型”曲面理论一样，嵌入满足如下性质: 对 $\mathcal{M}_{sw}$ 中每一点 z^M ，目标空间超 Vielbein 的拉回

$$E^{\underline{a}}(Z(z)) = dz^M \partial_M Z^{\underline{M}}_{L} E^{\underline{a}}_{\underline{M}}(Z(z)) \tag{119}$$

can be brought, by a local $SO(1,10)$ Lorentz transformation $E^{\underline{a}} \to E^{\underline{a}} u_{\underline{a}}{}^{\underline{b}}$ with a matrix

可通过一个带有矩阵的局域 $SO(1,10)$ 洛伦兹变换 $E^{\underline{a}} \to E^{\underline{a}} u_{\underline{a}}{}^{\underline{b}}$

$$u_{\underline{a}}{}^{\underline{b}}(z) = \left(u_{\underline{a}}{}^{a}(z), u_{\underline{a}}{}^{i}(z)\right) \;\; a = 0,1\ldots,p;\;\; i = 1,\ldots,10-p, \tag{120}$$

to the frame in which the components of $E^{\underline{a}}$ orthogonal to the tangent space of $\mathcal{M}_{sw}$ are zero

变换到这样一个标架: 其中 $E^{\underline{a}}$ 正交于 $\mathcal{M}_{sw}$ 切空间的分量为零

$$E^{i} \;:=\; E^{\underline{a}} u_{\underline{a}}{}^{i} = 0, \tag{121}$$

while the components of $E^{\underline{a}}$ along the tangent space of $\mathcal{M}_{sw}$ are identified with the worldvolume vector supervielbein $e^a(z)$

而 $E^{\underline{a}}$ 沿 $\mathcal{M}_{sw}$ 切空间的分量等同于世界体积矢量超 Vielbein $e^a(z)$

$$E^{a} \;:=\; E^{\underline{b}} u_{\underline{b}}{}^{a} = e^{a}(z). \tag{122}$$

Notice the different nature of the lower and the upper indices of the components of the matrix $u_b{}^{\underline{a}}$. The lower index is associated with the vector representation of the $SO(1,10)$ Lorentz group, while the upper index is the cumulative index of the direct product of the vector representations of $SO(1,p) \times SO(D-p-1)$. To simplify the presentation, we will not make a notational distinction between them, and the meaning of the indices will be clear from the context.

请注意矩阵 $u_b{}^{\underline{a}}$ 分量的上下指标性质不同: 下指标对应 $SO(1,10)$ 洛伦兹群的矢量表示，上指标则是 $SO(1,p) \times SO(D-p-1)$ 矢量表示直积的累积指标。为简化表述，我们不对二者做记号区分，指标的含义可从上下文明确看出。

Performing the inverse Lorentz transformation of (121) and (122) with the matrix

用该矩阵对 (121) 和 (122) 做逆洛伦兹变换

$$u_{\underline{b}}^{-1\underline{a}}(z) = (u_a{}^{\underline{a}}, u_i{}^{\underline{a}}) \;\; u_a{}^{\underline{a}} \;:=\; \eta_{ab}\eta^{\underline{ab}} u_{\underline{b}}{}^{b}(z),\;\; u_i{}^{\underline{a}} \;:=\; -\delta_{ij}\eta^{\underline{ab}} u_{\underline{b}}{}^{j}(z)$$

(123)

we get back $E^{\underline{a}}$ in the worldvolume basis associated with the induced supervielbein e^{a} :

我们就得到了世界体积基下对应诱导超 Vielbein e^{a} 的 $E^{\underline{a}}$:

$$E^{\underline{a}}(Z(z)) = e^{a} u_{a}{}^{\underline{a}}. \tag{124}$$

In (123), the definition of the components of $u_{b}^{-1\underline{a}}$ reflects the orthogonality properties of the Lorentz matrices (112), and η_{ab} and $-\delta_{ij}$ are the components of the Minkowski metric $\eta_{\underline{ab}} = \left(\eta_{ab}, -\delta_{ij}\right)$ with the almost minus signature.

在 (123) 中，$u_{b}^{-1\underline{a}}$ 分量的定义体现了洛伦兹矩阵 (112) 的正交性，且 η_{ab} 和 $-\delta_{ij}$ 是近负号差的闵氏度规 $\eta_{\underline{ab}} = \left(\eta_{ab}, -\delta_{ij}\right)$ 的分量。

Comparing this expression with (116), we see that for the embedding of $\mathcal{M}_{sw}$ defined by (121) and (122) the superembedding condition (118) holds, and moreover the component $E_{a}^{\underline{a}}(Z(z))$ is part of the Lorentz matrix (123):

将该表达式与 (116) 对比可知，对由 (121) 和 (122) 定义的 $\mathcal{M}_{sw}$ 嵌入，超嵌入条件 (118) 成立，且分量 $E_{a}^{\underline{a}}(Z(z))$ 本身就是洛伦兹矩阵 (123) 的一部分:

$$E_{a}^{\underline{a}} = u_{a}{}^{\underline{a}}(z). \tag{125}$$

Note that in the case of the embedding of the pure bosonic surfaces into the target spaces, the embedding conditions (121) and (124) do not impose any restrictions on the induced worldvolume geometry, since they are purely conventional. Nevertheless, as it was discussed in [167], Equation (121) serves as a convenient starting point to reproduce the classical surface theory based on extrinsic geometry concepts and a related embedding approach to the bosonic strings [89, 110, 111, 168, 169].

注意，对于纯玻色曲面嵌入目标空间的情况，嵌入条件 (121) 和 (124) 不对诱导世界体积几何施加任何约束，因为它们纯粹是约定性的。尽管如此，正如文献 [167] 所讨论的，方程 (121) 是重现基于外几何概念的经典曲面理论、以及相关玻色弦嵌入方法的便利出发点 [89, 110, 111, 168, 169]。

We will call the components of the auxiliary worldvolume matrix superfields (120) and their inverse (123) the Lorentz vector harmonics. Note that the conditions (121) and (122) remain valid under the following local $SO(1,p) \times SO(10-p)$ transformations of $\left(u_{\underline{a}}{}^{a}, u_{\underline{a}}{}^{i}\right)$ and e^{a} :

我们将辅助世界体积矩阵超场 (120) 的分量及其逆 (123) 称为洛伦兹矢量调和。注意条件 (121) 和 (122) 在 $\left(u_{\underline{a}}{}^{a}, u_{\underline{a}}{}^{i}\right)$ 和 e^{a} 的如下局域 $SO(1,p) \times SO(10-p)$ 变换下保持不变:

$$\left(u'_{\underline{a}}{}^{a}, u'_{\underline{a}}{}^{i}\right) = \left(u_{\underline{a}}{}^{b} \mathcal{O}_{b}{}^{a}(z), u_{\underline{a}}{}^{j} \mathcal{O}_{j}{}^{i}(z)\right), \; e'^{a} = e^{b} \mathcal{O}_{b}{}^{a}(z). \tag{126}$$

$SO(1,p) \times SO(10-p)$ will be a local worldvolume symmetry of our construction. Since the $SO(1,10)$ Lorentz harmonics are defined modulo this symmetry, they parametrize a coset space $\frac{SO(1,D-1)}{SO(1,p)\times SO(D-p-1)}$, which explains their name.

$SO\left(1,p\right)\times SO\left(10-p\right)$ 将是我们构造中的一个局部世界体积对称性。由于 $SO\left(1,10\right)$ 洛伦兹调和是模去该对称性定义的，它们参数化了一个陪集空间 $\frac{SO(1,D-1)}{SO(1,p)\times SO(D-p-1)}$，这也解释了它们的命名。

This terminology stems from the seminal papers [170, 171] where the $SU\left(2\right)/U\left(1\right)$ and $SU\left(3\right)/\left[U\left(1\right)\times U\left(1\right)\right]$ harmonic superspaces were introduced and used to construct, for the first time, the off-shell superfield description of the $N=2$ and $N=3, D=4$ non-abelian supersymmetric Yang-Mills theories (see also the book [172]). The vector Lorentz harmonics (which can be also called moving frame variables; see below) were introduced in [28] under the name of light-cone harmonics and used their and in [29] to describe a superparticle model.

这一术语源自开创性论文 [170, 171]，其中引入了 $SU\left(2\right)/U\left(1\right)$ 和 $SU\left(3\right)/\left[U\left(1\right)\times U\left(1\right)\right]$ 调和超空间，并首次用其构造了 $N=2$ 和 $N=3, D=4$ 非阿贝尔超对称杨-米尔斯理论的离壳超场描述 (也可见专著 [172])。矢量洛伦兹调和 (也可称为活动标架变量，见下文) 在文献 [28] 中以光锥调和的名称引入，并在 [28] 和 [29] 中被用于描述超粒子模型。

Another name used for the vector Lorentz harmonics is moving frame variables. The reasons are that the matrix (120) describes Lorentz transformations and hence is composed of the orthogonal and normalized vectors (112), which have the property to be orthogonal and parallel to the bosonic directions of the worldvolume superspace

矢量洛伦兹调和的另一个名称是活动标架变量。原因在于式 (120) 的矩阵描述洛伦兹变换，因此由正交归一化矢量 (112) 构成，这些矢量满足与世界体积超空间的玻色方向正交和平行的性质。

$$u^{a\underline{c}}\left(z\right)u_{\underline{c}}{}^{b}\left(z\right)=\eta^{ab},\ u^{a\underline{c}}\left(z\right)u_{\underline{c}}{}^{i}\left(z\right)=0,\ u^{i\underline{c}}\left(z\right)u_{\underline{c}}{}^{j}\left(z\right)=-\delta^{ij}. \tag{127}$$

The Lorentz frame which they define changes from ”point” to ”point” of the worldsheet superspace in such a way that the above properties are preserved. In the pure bosonic limit, we can speak about the p-brane surface which moves in time, thus forming the worldvolume, and the frame moves together with the p-brane preserving its orientation with respect to the worldvolume. Hence the name the moving frame variables.

它们定义的洛伦兹标架在世界面超空间的不同“点”之间变化，同时保持上述性质不变。在纯玻色极限下，我们可以讨论随时间运动的 p 膜曲面，它由此形成世界体积，而标架随 p 膜一同运动，保持其相对于世界体积的定向不变。因此得名活动标架变量。

Let us now proceed with the definition of the induced fermionic supervielbein in $\mathcal{M}_{sw}$. To this end, let us note that as in the case of (the pullback of) the vector supervielbein (119), the fermionic supervielbein $E^{\underline{\alpha}}$ and its pullback are defined modulo a local transformation acting as the Majorana spinor representation of $SO\left(1,10\right)$ (or $\mathrm{Spin}\left(1,10\right)$). The parameters of this transformation form a 32×32 matrix $v_{\beta}{}^{\alpha}$ which is related to the matrix $u_{\underline{b}}{}^{\underline{a}}$, Eq. (120), in the vector representation of $SO\left(1,10\right)$ as follows:

现在我们来定义 $\mathcal{M}_{sw}$ 中诱导的费米子超 vielbein。为此我们注意到，和矢量超 vielbein(119)(的拉回) 的情况一样，费米子超 vielbein $E^{\underline{\alpha}}$ 及其拉回是模去 $SO\left(1,10\right)$ (或 $\mathrm{Spin}\left(1,10\right)$) 马约拉纳旋量表示下的局部变换定义的。该变换的参数构成 32×32 矩阵 $v_{\beta}{}^{\alpha}$ ，它与式 (120) 中的矩阵 $u_{\underline{b}}{}^{\underline{a}}$ 在 $SO\left(1,10\right)$ 的矢量表示中满足如下关系:

$$v\Gamma^{\underline{a}}v^T = \Gamma^{\underline{b}}u_{\underline{b}}{}^{\underline{a}} \Rightarrow \begin{cases} v\Gamma^{a}v^T = \Gamma^{\underline{b}}u_{\underline{b}}{}^{a}, \\ v\Gamma^{i}v^T = \Gamma^{\underline{b}}u_{\underline{b}}{}^{i} \end{cases} \tag{128}$$

The relation of $v_{\underline{\beta}}{}^{\underline{\alpha}}$ to the inverse matrix $u_{\underline{b}}^{-1\underline{a}}$ (123) is

> $v_{\underline{\beta}}{}^{\underline{\alpha}}$ 与逆矩阵 $u_{\underline{b}}^{-1\underline{a}}$ (123) 的关系为

$$v^T\Gamma^{\underline{a}}v = \Gamma^{\underline{b}}u_{\underline{b}}^{-1\underline{a}} = \Gamma^{b}u_{b}{}^{\underline{a}} + \Gamma^{i}u_{i}{}^{\underline{a}}. \tag{129}$$

The Lorentz group transformations also preserve the charge conjugation matrix $C_{\underline{\alpha\beta}}$ and its inverse $C^{\underline{\alpha\beta}}$. So the matrix $v_{\underline{\beta}}{}^{\underline{\alpha}}$ satisfies the conditions:

> 洛伦兹群变换也保持电荷共轭矩阵 $C_{\underline{\alpha\beta}}$ 及其逆 $C^{\underline{\alpha\beta}}$ 不变。因此矩阵 $v_{\underline{\beta}}{}^{\underline{\alpha}}$ 满足条件:

$$vCv^T = C, \; v^TC^{-1}v = C^{-1}. \tag{130}$$

These relations imply that the inverse matrix $v_{\beta}^{-1\underline{\alpha}}$ is related to $v_{\underline{\alpha}}{}^{\underline{\beta}}$ by a kind of transposition. The explicit expressions of the Dirac matrices and the charge conjugation matrix are given in the Appendices "Appendix A: $SO(1,2)\times SO(8)$ Invariant Representation for $D = 11$ Dirac Matrices and M2-Brane Lorentz Harmonics" and "Appendix B: $SO(1,5) \times SO(5)$ Invariant Representation for $D = 11$ Dirac Matrices and M5-Brane Lorentz Harmonics".

> 这些关系说明逆矩阵 $v_{\beta}^{-1\underline{\alpha}}$ 与 $v_{\underline{\alpha}}{}^{\underline{\beta}}$ 通过某种转置关系相联系。狄拉克矩阵和电荷共轭矩阵的显式表达式可见附录"附录 A: $SO(1,2)\times SO(8)$ 不变表示下的 $D = 11$ 狄拉克矩阵与 M2 膜洛伦兹调和"和"附录 B: $SO(1,5) \times SO(5)$ 不变表示下的 $D = 11$ 狄拉克矩阵与 M5 膜洛伦兹调和"。

So, the matrices v and u carry the same number of independent components which is equal to the dimension of $SO(1,10)$. This number is further reduced by the action of the gauge group $SO(1,p) \times SO(10-p)$. We will, therefore, call $v_{\alpha}{}^{\beta}$ the Lorentz spinor harmonics as in $[32-34, 38, 40, 41, 89]$. Another name, spinor moving frame variables, used for the spinor Lorentz harmonics in [36,38,40,68,69], reflects the fact that they provide a kind of square root of the moving frame vectors, i.e., the vector Lorentz harmonics, in the sense of Equations (128) and (129). In other words they are counterparts of $D = 4$ diades which are used to construct a light-like tetrad of the Newman-Penrose formalism of general relativity [173] by using the Cartan-Penrose representation for the light-like four-vector [174].

> 因此，矩阵 v 和 u 拥有相同数量的独立分量，该数目等于 $SO(1,10)$ 的维度。这个数目会在规范群 $SO(1,p)\times SO(10-p)$ 的作用下进一步减少。因此，我们仿照 $[32-34, 38, 40, 41, 89]$ 将 $v_{\alpha}{}^{\beta}$ 称为洛伦兹旋量调和量。文献 [36,38,40,68,69] 中还将旋量洛伦兹调和量称为旋量动标变量，这反映了一个事实: 根据式 (128) 和 (129)，它们提供了动标向量即向量洛伦兹调和量的一种平方根形式。换句话说，它们是 $D = 4$ 双矢的对应物；在广义相对论纽曼-彭罗斯形式中，人们利用卡坦-彭罗斯表示构造类光四矢量，进而得到类光标架 [173,174]，$D = 4$ 双矢正是该构造中用到的对象。

As in the case of the vector supervielbein, we can use $v_{\alpha}{}^{\beta}$ to associate certain components of the pullback of the spinor supervielbein:

和向量超 Vielbein 的情况一样，我们可以利用 $v_{\alpha}{}^{\beta}$ 将旋量超 Vielbein 拉回的特定分量关联为:

$$E^{\underline{\alpha}} = dz^{M}\partial_{M}Z^{\underline{M}}E_{\underline{M}}^{\underline{\alpha}}\left(Z\left(z\right)\right) = e^{a}E_{a}^{\underline{\alpha}} + e^{\alpha q}E_{\alpha q}^{\underline{\alpha}} \tag{131}$$

with the induced spinor supervielbein $e^{\alpha q}$ in the superworldvolume. To this end, as in (120), we split the index $\underline{\alpha}$ into two 16-dimensional sets of indices, one of which is associated with the worldvolume spinor indices αq , and another one labeling a complimentary spinor representation which depends on the dimension of the superworldvolume, namely,

关联到超世界体中的诱导旋量超 Vielbein $e^{\alpha q}$ 。为此，和式 (120) 中的处理一样，我们将指标 $\underline{\alpha}$ 拆分为两个 16 维的指标集合: 一个集合对应世界体旋量指标 αq ，另一个集合标记依赖于超世界体维度的互补旋量表示，即

$$v_{\underline{\beta}}{}^{\underline{\alpha}} = \left(v_{\underline{\beta}}{}^{\alpha q}, v_{\underline{\beta}\alpha\dot{q}}\right) \in \mathrm{Spin}\left(1,10\right), \begin{cases} \alpha,\beta = 1,2, \\ q,\dot{q} = 1,\ldots,8, \end{cases} \quad \text{for } M2 \text{ -brane,}$$

(132)

$$v_{\underline{\beta}}{}^{\underline{\alpha}} = \left(v_{\underline{\beta}}{}^{\alpha q}, v_{\underline{\beta}\alpha}^{\ \ q}\right) \in \mathrm{Spin}\left(1,10\right), \begin{cases} \alpha,\beta = 1,2,3,4, \\ q = 1,2,3,4, \end{cases} \quad \text{for M5-brane.}$$

(133)

We identify the projection of the pullback of (131) along $v_{\beta}{}^{\alpha q}$ with the (induced) worldvolume spinor supervielbein:

我们将 (131) 沿 $v_{\beta}{}^{\alpha q}$ 拉回的投影识别为 (诱导的) 世界体旋量超 Vielbein:

$$e^{\alpha q} = E^{\underline{\beta}}v_{\underline{\beta}}^{\alpha q}. \tag{134}$$

Note that with such an identification

请注意，按照该识别结果

$$E_{a}^{\underline{\alpha}}v_{\underline{\alpha}}{}^{\alpha q} = 0. \tag{135}$$

Thus the introduction of the Lorentz-harmonic variables allowed us to construct the basic objects, i.e., the supervielbeins (122) and (134) describing an induced geometry of the worldvolume superspace, and also to reformulate the superembed-ding condition (118) in the alternative form given by (121) and (122).

因此, 引入洛伦兹调和变量后, 我们得以构造描述世界体超空间诱导几何的基本对象, 即超 Vielbein (122) 和 (134)，还能将超嵌入条件 (118) 重新表述为 (121) 和 (122) 给出的替代形式。

Induced $SO(1,p)\times SO(10-p)$ Connections on $\mathcal{M}_{sw}$

$SO(1,p)\times SO(10-p)$ 上的诱导 $\mathcal{M}_{sw}$ 联络

We shall now study the consequences of the embedding conditions (121) and (122) by taking their external differential:

我们现在将通过对嵌入条件 (121) 和 (122) 取外微分来研究它们的推论:

$$dE^i = dE^{\underline{a}} u_{\underline{a}}{}^i + E^{\underline{a}} du_{\underline{a}}{}^i = 0, \tag{136}$$

$$dE^a = dE^{\underline{a}} u_{\underline{a}}{}^a + E^{\underline{a}} du_{\underline{a}}{}^a. \tag{137}$$

This will lead us to the definition of the $SO(1,p)\times SO(10-p)$ connections of the induced superworld-volume geometry.

这将引导我们给出诱导超世界体积几何的 $SO(1,p)\times SO(10-p)$ 联络定义。

In the flat target superspace, which we further assume, the supervielbeins are the Volkov-Akulov one-form (12) and $E^{\underline{\alpha}} = d\Theta^{\underline{\alpha}}$, and $dE^{\underline{a}}$ is given in (114). Then (136) takes the form:

在我们进一步假设的平坦目标超空间中，超 vielbein 是 Volkov-Akulov 一元形式 (12) 和 $E^{\underline{\alpha}} = d\Theta^{\underline{\alpha}}$ ，且 $dE^{\underline{a}}$ 由 (114) 给出。那么 (136) 可写为:

$$dE^i = -id\Theta^{\underline{\alpha}}\Gamma^{\underline{a}}_{\underline{\alpha\beta}} d\Theta^{\underline{\beta}} u_{\underline{a}}{}^i + E^{\underline{a}} du_{\underline{a}}{}^i = 0. \tag{138}$$

Let us now elaborate on the second term $E^{\underline{a}} du_{\underline{a}}{}^i$ of this equation. Using the orthogonality properties (112) of the Lorentz harmonics (120) and (123), and the embedding condition (122), we have:

现在我们来详细讨论该方程的第二项 $E^{\underline{a}} du_{\underline{a}}{}^i$ 。利用洛伦兹调和 (120) 和 (123) 的正交性 (112)，以及嵌入条件 (122)，我们可得:

$$E^{\underline{a}} du_{\underline{a}}{}^i = E^{\underline{b}} u_{\underline{b}}{}^{\underline{c}} u_{\underline{c}}{}^{-1\underline{a}} du_{\underline{a}}{}^i = e^a u_a{}^{\underline{a}} du_{\underline{a}}{}^i + E^j u_j{}^{\underline{a}} du_{\underline{a}}{}^i, \tag{139}$$

where the last term is actually zero due to the embedding condition (121).

其中最后一项因嵌入条件 (121) 实际上等于零。

We now notice that the quantities $u_a{}^{\underline{a}} du_{\underline{a}}{}^i$ and $u_j{}^{\underline{a}} du_{\underline{a}}{}^i$ in (139) are nothing but components of the $SO(1,10)$ Cartan form (111):

我们现在注意到，(139) 中的量 $u_a{}^{\underline{a}} du_{\underline{a}}{}^i$ 和 $u_j{}^{\underline{a}} du_{\underline{a}}{}^i$ 正是 $SO(1,10)$ 嘉当形式 (111) 的分量:

$$\Omega_a{}^i = u_a{}^{\underline{a}} du_{\underline{a}}{}^i = -\left(dE^{\underline{a}}_{a}\right) u_{\underline{a}}{}^i, \tag{140}$$

$$\Omega_j{}^i = u_j{}^{\underline{a}} du_{\underline{a}}{}^i. \tag{141}$$

(remember the relation (125)).

(记得关系式 (125))。

The one-form $\Omega_j{}^i$ takes values in the $SO(10-p)$ subgroup of $SO(1,10)$ and is associated with the connection of the $SO(10-p)$ gauge symmetry in $\mathcal{M}_{sw}$. We thus rewrite (138) as follows:

一元形式 $\Omega_j{}^i$ 取值于 $SO(1,10)$ 的 $SO(10-p)$ 子群，与 $\mathcal{M}_{sw}$ 中 $SO(10-p)$ 规范对称性的联络相关。因此我们将 (138) 改写如下:

$$\mathcal{D}E^i \ := \ dE^i - E^j\Omega_j{}^i = e^a\Omega_a{}^i - id\Theta^{\underline{\alpha}}\Gamma^{\underline{a}}_{\underline{\alpha\beta}}d\Theta^{\underline{\beta}}u_{\underline{a}}{}^i = 0, \tag{142}$$

where $\mathcal{D}$ is the covariant derivative which includes the connection $\Omega_j{}^i$ when it acts on the quantities that carry $SO(10-p)$ indices.

其中 $\mathcal{D}$ 是协变导数，当它作用在带 $SO(10-p)$ 指标的量上时，会包含联络 $\Omega_j{}^i$ 。

The decomposition of the form $\Omega_a{}^i$ in the basis of the worldvolume superviel-beins is

形式 $\Omega_a{}^i$ 在世界体积超 vielbein 基下的分解为

$$\Omega_a{}^i = e^{\alpha q}\Omega_{\alpha q}a^i + e^b\Omega_{ba}{}^i. \tag{143}$$

We see that the symmetric tensor component $\Omega_{(ba)}{}^i$ of the form $\Omega_a{}^i$ does not enter the relation (142) and hence remains arbitrary at this stage. The explicit expression for this component is

我们可以看到，形式 $\Omega_a{}^i$ 的对称张量分量 $\Omega_{(ba)}{}^i$ 并不出现在关系式 (142) 中，因此在现阶段它仍是任意的。该分量的显式表达式为

$$K_{ab}{}^i \ := \ \Omega_{(ab)}{}^i = -\nabla_{(a}E_{b)}{}^{\underline{c}}u_{\underline{c}}{}^i \tag{144}$$

One can recognize in $K_{ab}{}^i(z)$ a superfield generalization of the second fundamental form of the classical surface theory. In our context it characterizes the worldvolume superspace considered as a supersurface in the $D = 11$ target superspace. The dynamical equations of motion of the M2- and M5-branes will arise as algebraic conditions on the leading ($\eta = 0$) component of this superfield.

我们可以看出，$K_{ab}{}^i(z)$ 是经典曲面理论第二基本形式的超场推广。在本文的语境下，它描述了作为 $D = 11$ 目标超空间中超曲面的世界体积超空间。M2 膜和 M5 膜的动力学运动方程会作为对该超场领头 ($\eta = 0$) 分量的代数条件出现。

Let us elaborate on the form of the differential (137) of the embedding condition (122) in the same way as we did for (136). Thus, using (121) we get:

现在我们沿用处理 (136) 的方式，详细推导嵌入条件 (122) 的微分 (137) 的形式。利用 (121) 我们得到:

$$de^{a}=-id\Theta^{\underline{\alpha}}\Gamma^{\underline{a}}_{\underline{\alpha\beta}}d\Theta^{\underline{\beta}}u_{\underline{a}}{}^{a}+e^{b}u_{b}{}^{\underline{a}}du_{\underline{a}}{}^{a}, \tag{145}$$

where $u_{b}{}^{\underline{a}}du_{\underline{a}}{}^{a}$ can be identified with the induced $SO(1,p)$ spin connection in $\mathcal{M}_{sw}$

其中 $u_{b}{}^{\underline{a}}du_{\underline{a}}{}^{a}$ 可以等同于 $\mathcal{M}_{sw}$ 中的诱导 $SO(1,p)$ 自旋联络

$$\Omega_{b}{}^{a}=u_{b}{}^{\underline{a}}du_{\underline{a}}{}^{a}. \tag{146}$$

Then (137) takes the form of a constraint on the vector component of the torsion of $\mathcal{M}_{sw}$:

此时 (137) 表现为对 $\mathcal{M}_{sw}$ 挠率的矢量分量的约束，形式如下:

$$T^{a}=\mathcal{D}e^{a}\ :=\ de^{a}-e^{b}\Omega_{b}{}^{a}=-id\Theta^{\underline{\alpha}}\Gamma^{\underline{a}}_{\underline{\alpha\beta}}d\Theta^{\underline{\beta}}u_{\underline{a}}{}^{a}. \tag{147}$$

On the examples of the M2- and M5-brane, we will see that (147) is one of the essential constraints of the supergravity geometry on $\mathcal{M}_{sw}$.

以 M2 膜和 M5 膜为例，我们可以看到 (147) 是 $\mathcal{M}_{sw}$ 上超引力几何的基本约束之一。

To proceed further we should also define superworldvolume covariant derivatives of the Lorentz spinor harmonics $v_{\beta}{}^{\alpha}$. In view of their relation to the Lorentz vector harmonics (128) and (129), the corresponding Cartan forms are related as follows:

为进一步推进推导，我们还需要定义洛伦兹旋量调和 $v_{\beta}{}^{\alpha}$ 的超世界体积协变导数。结合其与洛伦兹矢量调和 (128) 和 (129) 的关系，相应嘉当形式的关系如下:

$$v^{-1}dv=\frac{1}{4}(u^{-1}du)^{\underline{ab}}\Gamma_{\underline{ab}}=\frac{1}{4}\Omega^{\underline{ab}}\Gamma_{\underline{ab}}$$

$$=\frac{1}{4}\Omega^{ab}\Gamma_{ab}+\frac{1}{4}\Omega^{ij}\Gamma^{ij}+\frac{1}{2}\Omega^{ai}\Gamma_{a}\Gamma_{i}. \tag{148}$$

Using this relation one defines the Spin $(1,p)\times$Spin $(D-p-1)$ -covariant derivative acting on the upper indices of $v_{\beta}{}^{\underline{\alpha}}$ split into Spin $(1,p)\times$ Spin$(D-\ p-1)$ indices as in (132) and (133)

利用该关系可以定义作用在 $v_{\beta}{}^{\underline{\alpha}}$ 上指标的 Spin $(1,p)\times$Spin $(D-p-1)$ 协变导数，这些指标如 (132) 和 (133) 所示拆分为 Spin $(1,p)\times$ Spin$(D-\ p-1)$ 指标

$$\mathcal{D}v\ :=\ dv-\frac{1}{4}v\Gamma_{ab}\Omega^{ab}-\frac{1}{4}v\Gamma^{ij}\Omega^{ij}=\frac{1}{2}v\Gamma_{a}\Gamma_{i}\Omega^{ai} \tag{149}$$

Note that the covariant derivative of the Lorentz spinor harmonics is expressed in terms of the covariant Cartan form (140). To write an explicit form of (149), we need an $SO(1,p)\times SO(D-p-1)$ invariant representation for the matrices $\Gamma^{\underline{a}}=\left(\Gamma^{a},\Gamma^{i}\right)$, which is p -dependent. For the M2- and M5-brane cases, they are given in Appendices "Appendix A: $SO(1,2) \times SO(8)$ Invariant Representation for $D = 11$ Dirac Matrices and M2-Brane Lorentz Harmonics" and "Appendix B: $SO(1,5) \times SO(5)$ Invariant Representation for $D = 11$ Dirac Matrices and M5-Brane Lorentz Harmonics".

注意，洛伦兹旋量调和的协变导数可由协变嘉当形式 (140) 表示。要写出 (149) 的显式形式，我们需要矩阵 $\Gamma^{\underline{a}} = \left(\Gamma^{a}, \Gamma^{i}\right)$ 的一个 $SO(1,p) \times SO(D-p-1)$ 不变表示，该表示依赖于 p 。对于 M2 膜和 M5 膜情形，相关结果分别见附录 A: $SO(1,2) \times SO(8)$ 对 $D = 11$ 狄拉克矩阵与 M2 膜洛伦兹调和的不变表示，以及附录 B: $SO(1,5) \times SO(5)$ 对 $D = 11$ 狄拉克矩阵与 M5 膜洛伦兹调和的不变表示。

M2-Brane

M2 膜

We will now show how the superembedding conditions produce the dynamical equations of motion of the M2-brane. Technical material on which the discussion is based is collected in "Appendix A: $SO(1,2) \times SO(8)$ Invariant Representation for $D = 11$ Dirac Matrices and M2-Brane Lorentz Harmonics."

我们现在将展示超嵌入条件如何导出 M2 膜的动力学运动方程。讨论所依据的技术材料收集在“附录 A: $SO(1,2) \times SO(8)$ $D = 11$ 狄拉克矩阵和 M2 膜洛伦兹调和量的不变表示”中。

The M2-brane superworldvolume is an $n = 8, d = 3$ superspace $\mathcal{M}_{sw}^{(3|16)}$, where n stands for the number of two-component Majorana spinors in $d = 2+1$ so that the total number of the superworldvolume fermionic directions is 16 . The geometry of $\mathcal{M}_{sw}^{(3|16)}$ induced by its embedding into flat $D = 11$ superspace is characterized by the worldvolume supervielbeins determined by the superembedding conditions (121), (122), and (134), and by the corresponding connections of the local $SO(1,2) \times SO(8)$ symmetry (141) and (146). By construction the $\mathcal{M}_{sw}^{(3|16)}$ geometry is that of an $n = 8, d = 3$ supergravity characterized by the torsion constraint (147) the r.h.s. of which is still to be elaborated.

M2 膜的超世界体积是一个 $n = 8, d = 3$ 超空间 $\mathcal{M}_{sw}^{(3|16)}$，其中 n 代表 $d = 2+1$ 中双分量马约拉纳旋量的数量，因此超世界体积的整体费米方向数为 16。其嵌入平坦 $D = 11$ 超空间诱导出的 $\mathcal{M}_{sw}^{(3|16)}$ 几何，由超嵌入条件 (121)、(122) 和 (134) 确定的世界体积超 vielbein，以及对应局部 $SO(1,2) \times SO(8)$ 对称性的联络 (141) 和 (146) 刻画。按构造，$\mathcal{M}_{sw}^{(3|16)}$ 几何就是由挠率约束 (147) 表征的 $n = 8, d = 3$ 超引力，该约束的右侧仍需进一步推导。

Let us now have a look at the structure of the projection of the pullback of the target-space spinorial supervielbein $E^{\underline{\alpha}} = d\Theta^{\underline{\alpha}}$ along the fermionic directions "orthogonal" to those of $\mathcal{M}_{sw}^{(3|16)}$. This projection is made by the Lorentz spinor harmonics $v_{\underline{\alpha}\alpha\dot{q}}$ (132) (Note that the index $\dot{q} = 1, \ldots, 8$ labels a spinor representation of $SO(8)$ which is different from the $SO(8)$ spinor representation labeled by the index $q = 1, \ldots, 8$ in (134) and from the $SO(8)$ vector representation labeled by the index $i = 1, \ldots, 8$ in (121). The three representations are related by triality manifested in the $SO(8)$ invariance of the matrices $\gamma^{i}_{q\dot{q}}$. See Appendix "Appendix A: $SO(1,2) \times SO(8)$ Invariant Representation for $D = 11$ Dirac Matrices and M2-Brane Lorentz Harmonics" for more details.). In the worldvolume supervielbein basis (122) and (134), we have:

现在我们来考察目标空间旋量超 vielbein $E^{\underline{\alpha}} = d\Theta^{\underline{\alpha}}$ 沿“正交”于 $\mathcal{M}_{sw}^{(3|16)}$ 方向的费米方向拉回的投影结构。该投影由洛伦兹旋量调和函数 $v_{\underline{\alpha}\alpha\dot{q}}$ 完成 (132)(注意: 指标 $\dot{q} = 1, \ldots, 8$ 标记的是 $SO(8)$ 的一种旋量表示，它不同于 (134) 中指标 $q = 1, \ldots, 8$ 标记的 $SO(8)$ 旋量表示，也不同于 (121) 中指标 $i = 1, \ldots, 8$ 标记的 $SO(8)$ 向量表示。这三种表示由矩阵 $\gamma^i_{q\dot{q}}$ 的 $SO(8)$ 不变性中体现的三重性联系。更多细节见附录“附录 A: $SO(1,2) \times SO(8)$ 不变表示下的 $D = 11$ 狄拉克矩阵与 M2 膜洛伦兹调和函数”。) 在世界体积超 vielbein 基 (122) 和 (134) 下，我们有:

$$E_{\alpha\dot{q}} \;:=\; d\Theta^{\underline{\beta}} v_{\underline{\beta}\alpha\dot{q}} = e^{\beta r} h_{\beta r\alpha\dot{q}} + e^b \psi_{b\alpha\dot{q}}, \tag{150}$$

where $h_{\alpha q\beta\dot{q}}(z)$ and $\psi_{b\beta\dot{q}}(z)$ are bosonic and fermionic superfields in $\mathcal{M}_{sw}^{(3,16)}$.

其中 $h_{\alpha q\beta\dot{q}}(z)$ 和 $\psi_{b\beta\dot{q}}(z)$ 是 $\mathcal{M}_{sw}^{(3,16)}$ 中的玻色子和费米子超场。

To find constraints on these superfields, let us elaborate on the consequence (142) of the superembedding condition (121). Using the decompositions (143) and (150), and the relation (202) between the spinor and vector Lorentz harmonics, we write (142) as follows:

为了找到这些超场满足的约束，我们来推导超嵌入条件 (121) 的推论 (142)。利用分解式 (143) 和 (150)，以及旋量与矢量洛伦兹调和之间的关系 (202)，我们将 (142) 写为如下形式:

$$e^a e^{\alpha q} \Omega_{\alpha q a}{}^i + e^a e^b \Omega_{ba}{}^i = -2ie^{\alpha q}\gamma^i_{q\dot{q}} E_{\alpha\dot{q}} = -2ie^{\beta r} e^{\alpha q}\gamma^i_{q\dot{q}} h_{\beta r\alpha\dot{q}} \tag{151}$$

$$+2ie^b e^{\alpha q}\gamma^i_{q\dot{q}} \psi_{b\alpha\dot{q}},$$

where $\gamma^i_{q\dot{q}}$ are $SO(8)$ counterparts of the Pauli matrices.

其中 $\gamma^i_{q\dot{q}}$ 是泡利矩阵在 $SO(8)$ 下的对应形式。

Comparing the components of the left- and the right-hand side of the above equation, we see that

对比上述方程左右两侧的分量，我们可得

$$\Omega^i_{[ab]} = 0,\; \Omega_{\alpha q a}{}^i = 2i\gamma^i_{q\dot{p}}\psi^a_{\alpha\dot{p}} \tag{152}$$

and

和

$$\gamma^i_{q\dot{q}} h_{\beta r\alpha\dot{q}} + \gamma^i_{r\dot{q}} h_{\alpha q\beta\dot{q}} = 0, \tag{153}$$

whose solution is trivial

其解是平凡的

$$h_{\beta r\alpha\dot{q}} = 0.$$

So, (150) simplifies to

因此，(150) 简化为

$$E_{\alpha\dot{q}} \ := \ d\Theta^{\underline{\beta}} v_{\underline{\beta}\alpha\dot{q}} = e^b \psi_{b\alpha\dot{q}}, \tag{154}$$

while, in view of (152), the covariant form (143) reduces to

同时，结合 (152)，协变形式 (143) 约化为

$$\Omega_a{}^i = 2ie^{\alpha q}\gamma^i_{q\dot{q}}\psi_{a\alpha\dot{q}} + e^b K^i_{ab}, \tag{155}$$

where the symmetric tensor K^i_{ab} was identified as the second fundamental form in (144), and

其中对称张量 K^i_{ab} 在式 (144) 中被定义为第二基本形式，且

$$\psi_{a\alpha\dot{q}} = E^{\underline{\alpha}}_b v_{\underline{\alpha}\alpha\dot{q}} = \nabla_b \Theta^{\underline{\alpha}} v_{\underline{\alpha}\alpha\dot{q}}. \tag{156}$$

Using Equation (154) and (201) of Appendix "Appendix A: $SO(1,2) \times SO(8)$ Invariant Representation for $D = 11$ Dirac Matrices and M2-Brane Lorentz Harmonics," we can find the explicit form of the $\mathcal{M}^{(3,16)}_{sw}$ torsion (137):

利用附录“附录 A: $SO(1,2) \times SO(8)$ 不变表示: $D = 11$ 狄拉克矩阵与 M2 膜洛伦兹调和”中的式 (154) 和 (201)，我们可以得到 $\mathcal{M}^{(3,16)}_{sw}$ 挠率 (137) 的显式形式:

$$T^a = \mathcal{D}e^a = -ie^{\alpha q}e^{\beta q}\gamma^a_{\alpha\beta} - ie^b e^c \psi_{b\alpha\dot{q}}\gamma^{a\alpha\beta}\psi_{c\beta\dot{r}}\delta^{\dot{q}\dot{r}}, \tag{157}$$

which contains the basic essential constraint $T^a_{\alpha\beta} = -2i\gamma^a_{\alpha\beta}$ of the supergravity theory.

其中包含超引力理论的基本核心约束 $T^a_{\alpha\beta} = -2i\gamma^a_{\alpha\beta}$。

Now let us take the worldvolume covariant derivative of the left- and the righthand side of (154). The derivative (acting from the right) on the left-hand side reads:

现在我们对式 (154) 左右两侧取世界体积协变导数。作用在左侧的导数 (从右侧作用) 形式为:

$$\mathcal{D}\left(d\Theta^{\underline{\beta}} v_{\underline{\beta}\alpha\dot{q}}\right) = d\Theta^{\underline{\beta}}\mathcal{D}v_{\underline{\beta}\alpha\dot{q}} = -\frac{1}{2}e^{\beta p}\Omega_a{}^i\gamma^a_{\alpha\beta}\gamma^i_{p\dot{q}}$$

$$= \left(ie^{\beta p}e^{\delta q}\gamma^i_{q\dot{q}}\psi_{a\delta\dot{q}} - \frac{1}{2}e^{\beta p}e^a K^i_{ab}\right)\gamma^a_{\alpha\beta}\gamma^i_{p\dot{q}} \tag{158}$$

where we used that $e^{\alpha q} = d\Theta^{\underline{\alpha}} v^{\alpha q}_{\underline{\alpha}}$ and the expression for the covariant derivative of the spinorial harmonics $v_{\alpha,\alpha\dot{q}}$ (see (205)) derived from (149) with the use of the gamma-matrix representation given in Appendix

”Appendix A: $SO(1,2)\times SO(8)$ Invariant Representation for $D=11$ Dirac Matrices and M2-Brane Lorentz Harmonics.” In the last step, we substituted the explicit expression (155) for $\Omega_a{}^i$.

这里我们利用了 $e^{\alpha q}=d\Theta^{\underline{\alpha}}v_{\underline{\alpha}}^{\alpha q}$ ，以及由式 (149) 结合附录“附录 A: $SO(1,2)\times SO(8)$ 不变表示: $D=11$ 狄拉克矩阵与 M2 膜洛伦兹调和”给出的伽马矩阵表示推导得到的旋量调和 $v_{\alpha,\alpha\dot{q}}$ 协变导数表达式 (见式 (205))。最后一步我们代入了 $\Omega_a{}^i$ 的显式表达式 (155)。

The covariant derivative of the right-hand side of (154), with taking into account (157), gives:

考虑到式 (157)，对式 (154) 右侧取协变导数可得:

$$\mathcal{D}\left(e^b\psi_{b\alpha\dot{q}}\right)=-2ie^{\beta p}e^{\delta p}\gamma^b_{\beta\delta}\psi_{b\alpha\dot{q}}+ie^be^c\left(\psi_b\gamma^a\psi_c\right)\psi_{a\alpha\dot{q}}+e^b\mathcal{D}\psi_{b\alpha\dot{q}} \tag{159}$$

We should now equate the right-hand sides of (158) and (159). For the components of $e^{\beta p}\wedge e^{\delta q}$, we get:

现在我们需要令式 (158) 和式 (159) 的右侧相等。针对 $e^{\beta p}\wedge e^{\delta q}$ 分量，我们得到:

$$\gamma^i_{p\dot{q}}\gamma^i_{q\dot{p}}\gamma^a_{\alpha\beta}\psi_{a\gamma\dot{p}}+\gamma^i_{q\dot{q}}\gamma^i_{p\dot{p}}\gamma^a_{\alpha\gamma}\psi_{a\beta\dot{p}}=-2\delta_{pq}\psi_{a\alpha\dot{q}}. \tag{160}$$

The only consequence of this equation is $\gamma^{a\beta}_{\alpha}\psi_{a\beta\dot{q}}=0$ which in view of the explicit form (156) of $\psi_{a\beta\dot{q}}$ is a first-order Dirac-like differential equation for the superfield $\Theta^{\underline{\alpha}}$:

该方程唯一的推论是 $\gamma^{a\beta}_{\alpha}\psi_{a\beta\dot{q}}=0$ ，结合 $\psi_{a\beta\dot{q}}$ 的显式形式 (156)，可知这是超场 $\Theta^{\underline{\alpha}}$ 的一阶类狄拉克微分方程:

$$\gamma^{a\beta}_{\alpha}\nabla_a\Theta^{\underline{\alpha}}v_{\underline{\alpha}\beta\dot{q}}=0. \tag{161}$$

At $\eta=0$ this equation coincides with the physical equations of motion of the fermionic embedding coordinates $\theta^\alpha(\xi)$ of the M2-brane, while at higher orders it does not produce any further restrictions on the independent dynamical variables of M2.

在 $\eta=0$ 处，该方程与 M2 膜费米嵌入坐标 $\theta^\alpha(\xi)$ 的物理运动方程完全一致，而在更高阶它不会对 M2 膜的独立动力学变量给出任何额外约束。

To obtain the bosonic field equations of the M2-brane, let us equate the components of $e^b\wedge e^{\beta p}$ in (158) and (159). We thus get:

为得到 M2 膜的玻色场方程，我们令式 (158) 和式 (159) 中 $e^b\wedge e^{\beta p}$ 的分量相等，于是得到:

$$\frac{1}{2}\gamma^i_{p\dot{q}}\gamma^a_{\alpha\beta}K_{ab}{}^i=\mathcal{D}_{\beta p}\psi_{b\alpha\dot{q}}. \tag{162}$$

Contracting this equation with $\gamma^{b\gamma\alpha}$ one gets

将该方程与 $\gamma^{b\gamma\alpha}$ 缩并可得

$$\gamma^i_{p\dot{q}} K_a{}^{ai} \delta_\beta{}^\gamma = 2\mathcal{D}_{\beta p} \left(\gamma^a \psi_{a\dot{q}}\right)^\gamma . \tag{163}$$

The right-hand side of the above equation vanishes due to the fermionic equation of motion (161), and we arrive at the bosonic equations of motion of the M2-brane:

由于费米运动方程 (161)，上述方程的右侧为零，我们最终得到 M2 膜的玻色运动方程:

$$K_a{}^{ai} := -\nabla^a E_a{}^{\underline{b}} u_{\underline{b}}{}^i = 0, \; E_a^{\underline{b}} = \nabla_a X^{\underline{b}} - i\nabla_a \Theta \Gamma^{\underline{a}} \Theta. \tag{164}$$

To demonstrate a contact with the standard (Green-Schwarz-like) formulation of the supermembrane [6, 7] , let us consider the leading $\eta = 0$ component of (164) and formally set to zero the fermionic components of the supervielbein e_α^M , in particular $e_\alpha^\mu|_{\eta=0} = 0$. Then (164) reduces to differential equations on ordinary worldvolume fields $x^{\underline{a}}(\xi)$ and $\theta^{\underline{\alpha}}(\xi)$ which can be rewritten with the use of the induced worldvolume metric $g_{mn}(\xi) = e_m{}^a e_{an} = E_m{}^{\underline{a}} E_{n\underline{a}}$ (where now $E_m^{\underline{a}} = \; \partial_m x^{\underline{a}} - i\partial_m \theta \Gamma^{\underline{a}} \theta$) as follows:

为了证明其与超膜的标准 (格林-施瓦茨型) 表述 [6, 7] 一致，我们来考虑 (164) 的主导 $\eta = 0$ 分量，形式上令超 vielbein e_α^M 的费米分量为零，特别是 $e_\alpha^\mu|_{\eta=0} = 0$ 。此时 (164) 约化为普通世界体积场 $x^{\underline{a}}(\xi)$ 和 $\theta^{\underline{\alpha}}(\xi)$ 的微分方程，利用诱导世界体积度规 $g_{mn}(\xi) = e_m{}^a e_{an} = E_m{}^{\underline{a}} E_{n\underline{a}}$ (其中此时 $E_m^{\underline{a}} = \; \partial_m x^{\underline{a}} - i\partial_m \theta \Gamma^{\underline{a}} \theta$)) 可将其改写为如下形式:

$$\nabla^a E_a{}^{\underline{b}} u_{\underline{b}}{}^i = \frac{1}{\sqrt{|g|}} \partial_m \left(\sqrt{|g|} g^{mn} E_n{}^{\underline{b}}\right) u_{\underline{b}}{}^i = 0. \tag{165}$$

The equation (165) is the projection along $u_{\underline{a}}{}^i(\xi)$ of the GS supermembrane equation of motion:

方程 (165) 是 GS 超膜运动方程沿 $u_{\underline{a}}{}^i(\xi)$ 的投影:

$$\partial_m \left(\sqrt{|g|} g^{mn} E_n \frac{b}{2}\right) = 0. \tag{166}$$

The other projection of this equation which is tangential to the worldvolume, $\partial_m \left(\sqrt{|g|} g^{mn} E_n{}^{\underline{b}}\right) u_{\underline{b}}{}^b = 0$, or equivalently

该方程切于世界体积的另一投影，即 $\partial_m \left(\sqrt{|g|} g^{mn} E_n{}^{\underline{b}}\right) u_{\underline{b}}{}^b = 0$ ，或等价地

$$\partial_m \left(\sqrt{|g|} g^{mn} E_n{}^{\underline{b}}\right) u_{\underline{b}}{}^b e_{bl} = 0, \tag{167}$$

is satisfied identically. This is a Noether identity reflecting the reparametrization invariance of the supermembrane action. Indeed, remembering that $u_a{}^{\underline{a}} = E_a^{\underline{a}}$ (see Eq. (125)) and that $u_{\underline{a}}{}^a = \eta^{ab} \eta_{\underline{ab}} u_b{}^{\underline{b}}$, we have $u_{\underline{b}}{}^b e_{bl} = E_l^{\underline{a}} \eta_{\underline{ab}}$. So (167) takes the form:

是恒成立的。这是反映超膜作用量重参数化不变性的诺特恒等式。实际上，记住 $u_a{}^{\underline{a}} = E_a^{\underline{a}}$ (见式 (125)) 且 $u_{\underline{a}}{}^a = \eta^{ab} \eta_{\underline{ab}} u_b{}^{\underline{b}}$ ，我们可得 $u_{\underline{b}}{}^b e_{bl} = E_l^{\underline{a}} \eta_{\underline{ab}}$ 。因此 (167) 可写为:

$$\partial_m \left(\sqrt{|g|} g^{mn} E_n{}^{\underline{b}}\right) E_l^{\underline{a}} \eta_{\underline{ab}} = 0. \tag{168}$$

Integrating this equation by parts and using the torsion constraint (114), we get:

对该方程分部积分并利用挠率约束 (114)，我们得到:

$$\partial_l \left(\sqrt{|g|}\right) - \sqrt{|g|} g^{mn} E_n \frac{b}{2} \eta_{\underline{ab}} \partial_l E_m^{\underline{a}} + i\sqrt{|g|} g^{mn} E_n^{\underline{a}} \partial_m \Theta \Gamma_{\underline{a}} \partial_l \Theta = 0. \tag{169}$$

Since $g_{mn}(\xi) = E_m{}^{\underline{a}} E_{n\underline{a}}$, the first two terms on the left-hand side of this equation cancel each other, while the third term is proportional to the fermionic equations of motion and hence also vanishes.

由于 $g_{mn}(\xi) = E_m{}^{\underline{a}} E_{n\underline{a}}$ ，方程左侧前两项相互抵消，而第三项与费米运动方程成正比，因此也等于零。

Finally, let us confront the $e^b e^c$ components of (158) and (159). In (158) this component is absent; hence the corresponding component in (159) must vanish, i.e.,

最后，我们来对比 (158) 和 (159) 的 $e^b e^c$ 分量。该分量在 (158) 中不存在，因此它在 (159) 中的对应分量必须为零，即:

$$\mathcal{D}_{[c]\psi b\alpha\dot{q}} = -i\psi_c \tilde{\gamma}^a \psi_b \psi_{a\alpha\dot{q}}. \tag{170}$$

One can check that this is indeed the case. To this end one should take the covariant derivative of (156), antisymmetrize it with respect to the indices $[a, b]$, and then use the form of the torsion (157), the expression of the covariant derivative of $v_{\underline{\alpha}\alpha\dot{q}}$ in (205), and the condition (135).

可以验证事实确实如此。为此需要对 (156) 取协变导数，对指标 $[a, b]$ 做反对称化，再利用 (157) 的挠率形式、(205) 中 $v_{\underline{\alpha}\alpha\dot{q}}$ 协变导数的表达式，以及条件 (135)。

We have thus shown that the superembedding conditions imposed on the embedding of the $n = 8, d = 3$ worldvolume superspace into the $D = 11$ superspace completely determine the on-shell dynamics of the M2-brane.

我们由此证明，施加在 $n = 8, d = 3$ 世界体积超空间嵌入 $D = 11$ 超空间上的超嵌入条件，完全确定了 M2 膜的在壳动力学。

Kappa-Symmetry from Worldvolume Superdiffeomorphisms

来自世界体积超微分同胚的卡帕对称性

As we have mentioned, the superembedding construction is invariant under the superdiffeomorphisms of the worldvolume supercoordinates $z^M \to z'^M(Z)$. The M2-brane coordinate functions $Z^{\underline{M}}(z) = \left(X^{\underline{m}}(z), \Theta^{\underline{\mu}}(z)\right)$ transform under these diffeomorphisms as scalar worldvolume superfields. Their infinitesimal transformations are

如我们所述，超嵌入构造在世界体积超坐标 $z^M \to z'^M(Z)$ 的超微分同胚下不变。M2 膜坐标函数 $Z^{\underline{M}}(z) = \left(X^{\underline{m}}(z), \Theta^{\underline{\mu}}(z)\right)$ 作为标量世界体积超场在这些微分同胚下变换。它们的无穷小变换为

$$\delta Z^{\underline{M}}(z) = \delta z^M \partial_M Z^{\underline{M}}(z) = i_\delta e^A \nabla_A Z^{\underline{M}}(z).$$

We can project these transformations along the components $E_{\underline{M}}^{\underline{A}}$ of the target space supervielbein:

我们可以将这些变换沿目标空间超 Vielbein 的分量 $E_{\underline{M}}^{\underline{A}}$ 做投影:

$$i_\delta E^{\underline{a}} = i_\delta e^A \nabla_A Z^{\underline{M}} E_{\underline{M}}^{\underline{a}}, \tag{171}$$

$$i_\delta E^{\underline{\alpha}} = i_\delta e^A \nabla_A Z^{\underline{M}} E_{\underline{M}}^{\underline{\alpha}}. \tag{172}$$

Let us now restrict ourselves to the fermionic diffeomorphisms only, i.e., set $i_\delta e^a = 0$ and $i_\delta e^{\alpha q} = \kappa^{\alpha q}(z)$. Then, due to the superembedding condition (118), the equations (171) and (171) reduce to

现在我们仅考虑费米型微分同胚，即令 $i_\delta e^a = 0$ 和 $i_\delta e^{\alpha q} = \kappa^{\alpha q}(z)$。此时，由超嵌入条件 (118)，方程 (171) 和 (171) 约化为

$$i_\kappa E^{\underline{a}} = \kappa^{\alpha q} E_{\alpha q}^{\underline{a}} = 0, \tag{173}$$

and

和

$$i_\kappa E^{\underline{\alpha}} = \kappa^{\alpha q} \nabla_{\alpha q} Z^{\underline{M}} E_{\underline{M}}^{\underline{\alpha}} = \kappa^{\alpha q} v_{\alpha q}{}^{\underline{\alpha}}, \tag{174}$$

where in the last equality we used the fact that $E_{\alpha q}^{\underline{\alpha}} = v_{\alpha q}{}^{\underline{\alpha}}$, as a consequence of the superembedding conditions (134) and (154).

其中在最后一个等式中我们用到了由超嵌入条件 (134) 和 (154) 得到的结论 $E_{\alpha q}^{\underline{\alpha}} = v_{\alpha q}{}^{\underline{\alpha}}$。

Let us now note that since the parameter $\kappa^{\alpha q}(z)$ is arbitrary, we can equivalently express it as the $v_{\underline{\beta}}{}^{\alpha q}$ projection of a $D = 11$ spinor superfield parameter $\kappa^{\underline{\beta}}(z)$:

现在注意到由于参数 $\kappa^{\alpha q}(z)$ 是任意的，我们可以将其等价表示为 $D = 11$ 旋量超场参数 $\kappa^{\underline{\beta}}(z)$ 的 $v_{\underline{\beta}}{}^{\alpha q}$ 投影:

$$\kappa^{\alpha q}(z) = \kappa \underline{\beta}(z) v_{\underline{\beta}}^{\alpha q}(z).$$

Then the fermionic variation (174) takes the form

此时费米变分 (174) 可以写为

$$i_\kappa E^{\underline{\alpha}} = \kappa^{\underline{\beta}} \left(v_{\underline{\beta}}{}^{\alpha q} v_{\alpha q}{}^{\underline{\alpha}} \right) = \frac{1}{2} \kappa^{\underline{\beta}} \left(1 - \overline{\Gamma} \right)_{\underline{\beta}}{}^{\underline{\alpha}}, \tag{175}$$

where we used the Lorentz-harmonic relations (203) and (204).

其中我们用到了洛伦兹调和关系 (203) 和 (204)。

At $\eta^{\alpha q} = 0$, the fermionic diffeomorphism variations (173) and (175) are equivalent to the kappa-symmetry transformations of the M2-brane coordinate functions $x^{\underline{a}}(\xi)$ and $\theta^{\underline{\alpha}}(\xi)$ which leave invariant the M2-brane action of [6,7].

在 $\eta^{\alpha q} = 0$ 处，费米微分同胚变分 (173) 和 (175) 等价于 M2 膜坐标函数 $x^{\underline{a}}(\xi)$ 和 $\theta^{\underline{\alpha}}(\xi)$ 的卡帕对称性变换，该变换保持文献 [6,7] 中 M2 膜的作用量不变。

M5-Brane

M5-膜

The on-shell dynamics of the M5-brane is also fixed by the superembedding equation (118) [91], or by its equivalent form (121) and (122). It is amazing that the superembedding condition also contains nonlinear equations of motion of a chiral two-form gauge field living on the M5-brane, whose three-form field strength is self-dual. We will demonstrate this fact in this section. Some technical relations required to follow the discussion are given in "Appendix B: $SO(1,5) \times SO(5)$ Invariant Representation for $D = 11$ Dirac Matrices and M5-Brane Lorentz Harmonics."

M5-膜的在壳动力学同样由超嵌入方程 (118)[91]，或其等价形式 (121) 与 (122) 确定。令人惊奇的是，超嵌入条件也包含了 M5-膜上存在的手征二形式规范场的非线性运动方程，该规范场的三形式场强是自对偶的。我们将在本节证明这一结论。支撑后续讨论所需的部分技术关系见“附录 B: $SO(1,5) \times SO(5)$ 不变表示下的 $D = 11$ 狄拉克矩阵与 M5-膜洛伦兹调和”。

The M5-brane superworldvolume is an $n = (2,0)$, $d = 6$ superspace $\mathcal{M}_{sw}^{(6|16)}$, where n here traditionally stands for the number of eight-component spinors in $d = 5+1$ so that the total number of the superworldvolume fermionic directions is 16. The geometry of $\mathcal{M}_{sw}^{(3|16)}$ induced by its embedding into flat $D = 11$ superspace is characterized by the worldvolume supervielbeins determined by the superembed-ding conditions (121), (122), and (134), and by the corresponding connections of the local $SO(1,5) \times SO(5)$ symmetry (141) and (146). By construction the $\mathcal{M}_{sw}^{(3|16)}$ geometry is that of an $n = (2,0)$, $d = 6$ supergravity characterized by the torsion constraint (147). This supergravity is composite (induced by superembedding) and thus does not carry independent dynamical degrees of freedom.

M5-膜的超世界体是一个 $n = (2,0)$, $d = 6$ 超空间 $\mathcal{M}_{sw}^{(6|16)}$，其中 n 在此习惯上代表 $d = 5+1$ 中八分量旋量的数量，因此超世界体费米方向的总数为 16。由嵌入平直 $D = 11$ 超空间诱导得到的 $\mathcal{M}_{sw}^{(3|16)}$ 几何，其特征由超嵌入条件 (121)、(122) 和 (134) 确定的世界体超维伊比因，以及对应的定域 $SO(1,5) \times SO(5)$ 对称性的联络 (141) 和 (146) 刻画。按构造，$\mathcal{M}_{sw}^{(3|16)}$ 几何是由挠率约束 (147) 表征的 $n = (2,0)$, $d = 6$ 超引力。该超引力是复合的 (由超嵌入诱导产生)，因此不携带独立的动力学自由度。

The projections of the pullback of the fermionic supervielbein $E^{\underline{\alpha}} = d\Theta^{\underline{\alpha}}$ on the Lorentz-harmonic superfields (133) are

费米子超维伊比因 $E^{\underline{\alpha}} = d\Theta^{\underline{\alpha}}$ 在洛伦兹调和超场 (133) 上的拉回投影为

$$E^{\alpha q} := E^{\underline{\beta}} v_{\underline{\beta}}^{\alpha q} = e^{\alpha q}, \tag{176}$$

$$E_{\beta}^{q} := E^{\underline{\alpha}} v_{\underline{\alpha}\beta}^{q} = e^{\alpha q} h_{\alpha p} \beta^{q} + e^{b} \psi_{b\beta}{}^{q}. \tag{177}$$

Note that the upper and lower spinor indices α of Spin $(1,5)$ are not related to each other by a charge conjugation matrix (see "Appendix B: $SO(1,5) \times SO(5)$ Invariant Representation for $D = 11$ Dirac Matrices and M5-Brane Lorentz Harmonics").

请注意, Spin $(1,5)$ 的上下旋量指标 α 无法通过电荷共轭矩阵相互联系 (见“附录 B: $SO(1,5)\times SO(5)$ 不变表示下的 $D = 11$ 狄拉克矩阵与 M5-膜洛伦兹调和”)。

Using Eqs. (176) and (177), and the decomposition of unity in terms of the Lorentz harmonics, $\delta_{\underline{\beta}}{}^{\underline{\alpha}} = v_{\underline{\beta}}{}^{\alpha q} v_{\alpha q}{}^{\underline{\alpha}} + v_{\underline{\beta}\alpha}{}^{q} v_{q}^{\alpha\underline{\alpha}}$, we get:

利用式 (176)、(177), 以及洛伦兹调和的单位分解 $\delta_{\underline{\beta}}{}^{\underline{\alpha}} = v_{\underline{\beta}}{}^{\alpha q} v_{\alpha q}{}^{\underline{\alpha}} + v_{\underline{\beta}\alpha}{}^{q} v_{q}^{\alpha\underline{\alpha}}$, 我们得到:

$$d\Theta^{\underline{\alpha}} = e^{\beta q} E_{\beta q}{}^{\underline{\alpha}} + e^{a} \psi_{a\beta}{}^{p} v_{p}{}^{\beta\underline{\alpha}},\ E_{\beta q}{}^{\underline{\alpha}} = v_{\beta p}{}^{\underline{\alpha}} + h_{\beta q \gamma}{}^{p} v_{p}{}^{\gamma\underline{\alpha}}, \tag{178}$$

Let us now consider the consequences of (142). We have:

现在我们来讨论 (142) 的推论，我们有:

$$e^{a}\Omega_{a}{}^{i} = i d\Theta^{\underline{\alpha}} \Gamma^{a}_{\underline{\alpha}\underline{\beta}} d\Theta^{\underline{\beta}} u_{\underline{a}}{}^{i}$$

or, expliciently in the basis of $(e^{a}, e^{\alpha q})$

或者，在 $(e^{a}, e^{\alpha q})$ 基下显式写为

$$e^{a} e^{\alpha q} \Omega_{\alpha q} a^{i} + e^{a} e^{b} \Omega_{ba}{}^{i} = -2 e^{\alpha q} \gamma^{i}_{qr} E_{\alpha r} = -2 e^{\beta r} e^{\alpha q} \gamma^{i}_{qp} h_{\beta r} \alpha^{p} - 2 e^{b} e^{\alpha q} \gamma^{i}_{qr} \psi_{ba} \alpha^{r},$$

(179)

where to arrive at the right-hand side, we used the identities (215) and the form of (176) and (177). Since the left-hand side of the above equation does not have an $e^{\beta r} e^{\alpha q}$ component, from its right-hand side we must conclude that

其中为了得到右侧表达式，我们使用了恒等式 (215) 以及式 (176) 和 (177) 的形式。由于上述方程的左侧不含 $e^{\beta r} e^{\alpha q}$ 分量，我们从其右侧可推得

$$h_{\beta p}\alpha^r\gamma^i_{qr} + h_{\alpha q}\beta^r\gamma^i_{pr} = 0. \tag{180}$$

Using the properties (210)-(213) of γ^i_{qp} , we find that (180) implies

利用 γ^i_{qp} 的性质 (210)-(213)，我们发现 (180) 意味着

$$h_{\alpha p\beta}{}^q = h_{\alpha\beta}\delta_p{}^q,\ h_{\alpha\beta} = h_{\beta\alpha}. \tag{181}$$

In $d = 6$, the basis of the covariant symmetric spin tensor matrices is formed by the antisymmetric product of three matrices γ^a which is anti-self-dual (see (208)):

在 $d = 6$ 中，协变对称旋张量矩阵的基由三个矩阵 γ^a 的反自对偶反对称乘积构成 (见 (208)):

$$\gamma^{abc}_{\alpha\beta} = \gamma^{[a}_{\alpha\gamma}\tilde{\gamma}^{b|\gamma\delta}\gamma^{|c]}_{\delta\alpha},\ \gamma^{abc}_{\alpha\beta} = -\frac{1}{3!}\varepsilon^{abcdef}\left(\gamma_{def}\right)_{\alpha\beta}. \tag{182}$$

Therefore, we can express $h_{\alpha\beta}$ as follows:

因此，我们可以将 $h_{\alpha\beta}$ 表示为:

$$h_{\alpha\beta} = \frac{1}{3!}h_{abc}\gamma^{abc}_{\alpha\beta}, \tag{183}$$

where h_{abc} is an antisymmetric self-dual tensor

其中 h_{abc} 是反对称自对偶张量

$$h_{abc} = \frac{1}{3!}\varepsilon_{abcdef}h^{def}. \tag{184}$$

A useful identity, which follows from the self-duality properties of h^{abc} and gamma-matrix properties (208), is

由 h^{abc} 的自对偶性质和伽马矩阵性质 (208) 可得到一个有用的恒等式:

$$h_{\alpha\gamma}\tilde{\gamma}^{a\gamma\delta}h_{\delta\beta} = \gamma^b_{\alpha\beta}k_b{}^a,\ k_b{}^a = -2h_{bcd}h^{cda}. \tag{185}$$

We shall now show that the self-dual tensor h_{abc} is related to the field strength of a chiral two-form gauge field living in the M5-brane worldvolume superspace:

我们现在将说明自对偶张量 h_{abc} 与存在于 M5 膜世界体积超空间中的手征二形式规范场的场强相关:

$$b_2 = \frac{1}{2}dz^N dz^M b_{MN}(z) = \frac{1}{2}e^B e^A b_{AB}(z).$$

Using an analogy with worldvolume vector gauge fields in the Green-Schwarz-like formulations of the D-branes [141-143], the authors of [91] assumed that the field strength of b_2 has the following form (This assumption was later shown to be in agreement [151,175] with the chiral form field strengths appearing in the M5-brane action [113, 114].)

类比 D 膜格林-施瓦茨类表述中的世界体积矢量规范场 [141-143]，文献 [91] 的作者假设 b_2 的场强具有如下形式 (该假设后续被证明与 M5 膜作用量 [113, 114] 中出现的手征形式场强一致 [151,175]。)

$$H_3 := db_2 - C_3 = \frac{1}{3!} e^c e^b e^a H_{abc}, \tag{186}$$

It is constrained to have nonzero components only along the bosonic directions of $\mathcal{M}_{sw}^{(6|16)}$, which is a natural assumption prompted by the fact that h_{abc} carries only the vector indices.

它受限于仅沿 $\mathcal{M}_{sw}^{(6|16)}$ 的玻色方向存在非零分量，这是由 h_{abc} 仅携带矢量指标这一事实推出的自然假设。

In (186) C_3 is the pullback on $\mathcal{M}_{sw}^{(6|16)}$ of the three-form gauge superfield of 11D supergravity. The explicit form of C_3 in the flat 11D superspace is somewhat cumbersome, but it is not required for our discussion. We only need to know the form of its field strength $\mathcal{F}_4$, which is much simpler and manifestly supersymmetric:

式 (186) 中，C_3 是 11 维超引力三形式规范超场在 $\mathcal{M}_{sw}^{(6|16)}$ 上的拉回。C_3 在平直 11 维超空间中的显式形式较为繁琐，但我们的讨论并不需要它。我们仅需要知道其场强 $\mathcal{F}_4$ 的形式，该形式简单得多且具备明显的超对称性:

$$\mathcal{F}_4 = dC_3 = \frac{1}{4} E^{\underline{b}} E^{\underline{a}} E^{\underline{\alpha}} E^{\underline{\beta}} \Gamma_{\underline{ab\alpha\beta}}. \tag{187}$$

To relate H_3 to the self-dual tensor h_{abc} , we should study the Bianchi identity:

为建立 H_3 与自对偶张量 h_{abc} 的联系，我们应当研究比安基恒等式:

$$dH_3 = -\mathcal{F}_4 = -\frac{1}{4} E^{\underline{b}} E^{\underline{a}} E^{\underline{\alpha}} E^{\underline{\beta}} \Gamma_{\underline{ab\alpha\beta}}. \tag{188}$$

Due to the constraint (186) on H_3 , the left-hand side of this identity is

由 H_3 满足的约束 (186)，该恒等式的左侧为

$$dH_3 = \frac{1}{2} T^c e^b e^a H_{abc} + \frac{1}{3!} e^c e^b e^a \mathcal{D} H_{abc}, \tag{189}$$

where the superworldvolume torsion two-form T^c was defined in (147). Substituting (178) into (147), we get:

其中超世界体积挠率二形式 T^c 已在式 (147) 中定义。将 (178) 代入 (147)，我们得到:

$$T^a = \mathcal{D} e^a = -2 e^{\alpha q} \wedge e^{\beta p} C_{qp} \gamma^b_{\alpha\beta} (\delta_b{}^a - k_b{}^a) + 2 e^b \wedge e^{\alpha q} C_{qp} (\psi_b{}^p \tilde{\gamma}^a h)_\alpha$$

$$+2e^c \wedge e^b \psi_b^q \tilde{\gamma}^a \psi_c^p C_{qp}$$

If we now equate the $e^{\alpha q} e^{\beta r} e^b e^a$ component of (189) (which appears only in its first term) with the corresponding component of the right-hand side of (188), then, taking into account the explicit form of the pullbacks of the supervielbeins $E^{\underline{\alpha}}$ and $E^{\underline{a}}$ given in (124) and (178), and relations between the Lorentz harmonics given in

如果我们将式 (189) 的 $e^{\alpha q} e^{\beta r} e^b e^a$ 分量 (仅出现在其第一项中) 与式 (188) 右侧的对应分量相等，那么结合式 (124) 和 (178) 给出的超 Vielbein $E^{\underline{\alpha}}$ 和 $E^{\underline{a}}$ 的拉回显式形式，以及给出的洛伦兹调和间的关系

"Appendix B: $SO(1,5) \times SO(5)$ Invariant Representation for $D = 11$ Dirac Matrices and M5-Brane Lorentz Harmonics," one can show [91, 92] that H_{abc} and h_{abc} are related to each other in the following way:

" 附录 B: $SO(1,5) \times SO(5)$ 关于 $D = 11$ 狄拉克矩阵与 M5 膜洛伦兹调和的不变表示"，可以证明 [91, 92] 满足 H_{abc} 与 h_{abc} 存在如下关系:

$$m_a{}^d H_{bcd} = h_{abc}, \tag{190}$$

or

$$H_{acd} = m_a^{-1d} h_{dbc}, \tag{191}$$

where

其中

$$m_a{}^b = \delta_a{}^b - k_a{}^b = \delta_a{}^b + 2h_{acd}h^{bcd} \tag{192}$$

and m_a^{-1b} is its inverse

且 m_a^{-1b} 是它的逆

$$m^{-1}{}a^b = \frac{1}{1 - \frac{1}{9} k_c{}^d k_d{}^c} \left(\delta_a^b + k_a{}^b\right). \tag{193}$$

It can be then shown that the $e^a e^b e^c e^d$ component of (188) produces the second-order equation of motion of the chiral gauge field b_2 :

可以进一步证明，式 (188) 的 $e^a e^b e^c e^d$ 分量给出了手征规范场 b_2 的二阶运动方程:

$$\mathcal{D}_{[a} H_{bcd]} = 3! C_{qp} \left(\psi_{[a} \tilde{\gamma}^e \psi_b\right) H_{cd]e}. \tag{194}$$

Note that this equation is actually the Bianchi identity for (186).

注意该方程实际上就是式 (186) 的比安基恒等式。

Now, as in the case of the M2-brane, one can proceed with the analysis of the consequences of the superembedding conditions and derive the equations of motion of the M5-brane coordinate functions Θ^{α} and $X^{\underline{a}}(z)$ which have the following form (in flat $D = 11$ superspace):

现在，和 M2 膜的情况一样，我们可以继续分析超嵌入条件的推论，推导出 M5 膜坐标函数 Θ^{α} 和 $X^{\underline{a}}(z)$ 的运动方程，其形式如下 (在平直 $D = 11$ 超空间中):

$$m^{ba}\left(\nabla_a \Theta^{\underline{\alpha}}\right)\left(E_b^{\underline{a}}\Gamma_{\underline{a}}\right)_{\underline{\alpha\beta}} v_q^{\alpha\underline{\alpha}} = 0, \tag{195}$$

$$m^{bc} m_c^a \left(\nabla_a E_b^{\underline{a}}\right) u_{\underline{a}}^i = 0. \tag{196}$$

For further details of the derivation of the M5-brane equations from superembed-ding, see the original articles [91,92] and the review [108], and for their equivalence to the equations obtained from the M5-brane action, see [108, 112].

关于从超嵌入推导 M5 膜方程的更多细节，参见原始文献 [91,92] 与综述 [108]；关于它们与从 M5 膜作用量得到的方程的等价性，参见 [108, 112]。

Conclusion

结论

In this chapter we have described the main features of the superembedding approach, which has not only allowed one to explain and clarify various classical and quantum properties of superstring and superbrane theory but has also found applications in the construction and description of new superbrane models, and of field theories with partially broken supersymmetry, as well as for solving practical problems. For instance, it has been used to derive new solutions of M5-brane equations [176, 177], for calculating vertex operators in (M2-M5)-brane systems [178], constructing higher-order contributions to brane actions [80, 179], and the study of branes ending on branes [180, 181, 181, 182]. It was also used in search for hypothetical branes [183] and the description of exotic branes such as the heterotic 5-brane [184], for the derivation of the equations of motion of multiple Dp -brane systems [185, 186] and of a multiple M-wave (multiple M0-brane) system [187-189]. In [190-192] a generalization of the embedding approach to worldvolume superspaces with so-called boundary fermions was developed and used as the basis for the construction of a nonconventional action for multiple D-branes.

本章我们介绍了超嵌入方法的主要特征，该方法不仅能够解释和阐明超弦与超膜理论的多种经典、量子性质，还已应用于构建和描述新超膜模型、部分破缺超对称的场论，以及解决实际问题。例如，该方法已被用于推导 M5 膜方程的新解 [176, 177]，计算 (M2-M5) 膜系统中的顶点算子 [178]，构建膜作用量的高阶贡献项 [80, 179]， 以及研究膜终止于膜的问题 [180, 181, 181, 182]。该方法还被用于寻找假想膜 [183]，描述诸如杂化 5 膜之类的奇异膜 [184]，推导多重 Dp 膜系统的运动方程 [185, 186] 与多重 M 波 (多重 M0 膜) 系统的运动方程 [187-189]。在文献 [190-192] 中，嵌入方法被推广到带有所谓边界费米子的世界体积超空间，并作为构建多重 D 膜非传统作用量的基础。

One may expect the superembedding methods be also useful for other purposes, such as a unified (S-duality) description of fundamental and solitonic extended objects [163] and in the context of double field theory and exceptional field theories.

可以预期，超嵌入方法对其他研究方向也同样有用，例如对基本 extended 对象与孤子 extended 对象的 (S 对偶) 统一描述 [163]，以及在双场理论与例外场论的研究框架中。

Appendix A: $SO(1,2)\times SO(8)$ Invariant Representation for $D=11$ Dirac Matrices and M2-Brane Lorentz Harmonics

附录 A: $SO(1,2)\times SO(8)$ 下 D = $D=11$ 狄拉克矩阵与 M2 膜洛伦兹调和的不变表示

We use the following $SO(1,2)\times SO(8)$ invariant representations for 11D Dirac matrices and charge conjugation matrix:

我们对 11 维狄拉克矩阵和电荷共轭矩阵采用如下 $SO(1,2)\times SO(8)$ 不变表示:

$$(\Gamma^{\underline{a}})_{\underline{\alpha}}{}^{\underline{\beta}} \equiv \left(\Gamma^{a}, \Gamma^{i}\right),\ a=0,1,2,\ i=1,\dots,8,$$

$$(\Gamma^{a})_{\underline{\alpha}}{}^{\underline{\beta}} \equiv \left(\Gamma^{\underline{0}}, \Gamma^{\underline{9}}, \Gamma^{\underline{10}}\right) \equiv \left(\Gamma^{0}, \Gamma^{1}, \Gamma^{2}\right) = \begin{pmatrix} \gamma^{a\beta}_{\alpha}\delta_{qp} & 0 \\ 0 & \gamma^{a}{}_{\beta}{}^{\alpha}\delta_{\dot{q}\dot{p}} \end{pmatrix},$$

$$(\Gamma^{i})_{\underline{\alpha}}{}^{\underline{\beta}} \equiv \left(\Gamma^{1}, \dots, \Gamma^{8}\right) = \begin{pmatrix} 0 & -i\varepsilon_{\alpha\beta}\gamma^{i}_{q\dot{p}} \\ -i\varepsilon^{\alpha\beta}\tilde{\gamma}^{i}_{\dot{q}p} & 0 \end{pmatrix}$$

$$C^{\underline{\alpha\beta}} = -C^{\underline{\beta\alpha}} = \mathrm{diag}\left(i\varepsilon^{\alpha\beta}\delta_{qp}, i\varepsilon_{\alpha\beta}\delta_{\dot{q}\dot{p}}\right), C_{\underline{\alpha\beta}} = \mathrm{diag}\left(-i\varepsilon_{\alpha\beta}\delta_{qp}, -i\varepsilon^{\alpha\beta}\delta_{\dot{q}\dot{p}}\right).$$

(197)

Here $\gamma^{a}{}_{\alpha}{}^{\beta}$ are $SO(1,2)$ Dirac matrices and $\gamma^{i}_{q\dot{q}}$ are $SO(8)$ Pauli-like matrices (Clebsch-Gordan coefficients), which obey the following relations:

其中 $\gamma^{a}{}_{\alpha}{}^{\beta}$ 是 $SO(1,2)$ 狄拉克矩阵，$\gamma^{i}_{q\dot{q}}$ 是 $SO(8)$ 泡利型矩阵 (克莱布希-高登系数)，它们满足下述关系:

$$\gamma^a_{\alpha\beta} := -i\gamma^a{}_\alpha{}^\gamma\varepsilon_{\gamma\beta} = \gamma^a_{\beta\alpha} = \gamma^a_{(\alpha\beta)},\ \gamma_a^{\alpha\beta} := i\varepsilon^{\alpha\gamma}\gamma^a{}_\gamma{}^\beta = \gamma_a^{(\alpha\beta)},\ \varepsilon^{\alpha\gamma}\varepsilon_{\gamma\beta} = \delta^\alpha_\beta,$$

$$\gamma^{ab} = -i\varepsilon^{abc}\gamma_c, \gamma^a_{\alpha\beta}\gamma_a^{\gamma\delta} = 2\delta_{(\alpha}{}^\gamma\delta_{\beta)}{}^\delta,$$

(198)

$$\tilde{\gamma}^i_{\dot{p}q} := \gamma^i_{q\dot{p}},\ \gamma^i_{q\dot{p}}\gamma^j_{q\dot{p}} + \gamma^j_{q\dot{p}}\gamma^i_{q\dot{p}} = 2\delta^{ij}\delta_{qp},\ \gamma^i_{p\dot{q}}\gamma^j_{p\dot{p}} + \gamma^j_{p\dot{q}}\gamma^i_{p\dot{p}} = 2\delta^{ij}\delta_{\dot{q}\dot{p}},$$

$$\gamma^i_{q\dot{q}}\gamma^i_{p\dot{p}} = \delta_{qp}\delta_{\dot{q}\dot{p}} + \frac{1}{4}\gamma^{ij}_{qp}\tilde{\gamma}^{ij}_{\dot{q}\dot{p}} \Rightarrow \gamma^i_{(q|\dot{q}}\gamma^i_{|p)\dot{p}} = \delta_{qp}\delta_{\dot{q}\dot{p}} = \gamma^i_{q(\dot{q}|}\gamma^i_{p|\dot{p})}.$$

(199)

Both 11D and 3d Dirac matrices are imaginary in our mostly minus signature conventions:

在我们采用的负号为主的度规约定中，11 维和 3 维狄拉克矩阵均为虚矩阵:

$$\eta^{ab} = \mathrm{diag}\,(+,-,\dots,-)\,,\ \eta^{ab} = \mathrm{diag}\,(+,-,-)\,. \tag{200}$$

Using the above representation, the relations (128) and (129) between the Lorentz harmonic variables adapted to the embedding of the M2-brane worldvolume superspace are

利用上述表示，适配 M2 膜世界体积超曲面嵌入的洛伦兹调和变量之间的关系式 (128) 和 (129) 为

$$u^a_{\underline{b}}\Gamma^{\underline{b}}_{\underline{\alpha\beta}} = v^{\alpha q}_{\underline{\alpha}}(\gamma_a)_{\alpha\beta}v^{\beta q}_{\underline{\beta}} + v_{\underline{\alpha}\alpha\dot{q}}(\gamma_a)^{\alpha\beta}v_{\underline{\beta}\beta\dot{q}}, \tag{201}$$

$$u^i_{\underline{b}}\Gamma^{\underline{b}}_{\underline{\alpha\beta}} = -2v_{(\underline{\alpha}|}{}^{\alpha q}\gamma^i_{q\dot{q}}v_{|\underline{\beta})\alpha\dot{q}}. \tag{202}$$

The $SO(1,10)$ spinor indices $\underline{\alpha}$ and β are raised and lowered by the charge conjugation matrices (197).

$SO(1,10)$ 旋量指标 $\underline{\alpha}$ 和 β 由电荷共轭矩阵 (197) 进行升降操作。

The unity decomposition in terms of the Lorentz spinor harmonics is

洛伦兹旋量调和下的单位分解为

$$\delta_{\underline{\beta}}{}^{\underline{\alpha}} = v_{\underline{\beta}}{}^{\alpha q}v_{\alpha q}{}^{\underline{\alpha}} + v_{\underline{\beta}\alpha\dot{q}}v^{\underline{\alpha}\alpha\dot{q}}, \tag{203}$$

while

而

$$v_{\underline{\beta}\alpha\dot{q}}v^{\underline{\alpha}\alpha\dot{q}} - v_{\underline{\beta}}{}^{\alpha q}v_{\alpha q}{}^{\underline{\alpha}} = \frac{i}{3!}\varepsilon_{abc}\left(v_{\alpha q}{}^{\underline{\alpha}}\tilde{\gamma}^{abc\alpha\beta}v_{\beta q}{}^{\underline{\gamma}} + v^{\alpha\dot{q}\underline{\alpha}}\gamma^{abc}_{\alpha\beta}v^{\beta\dot{q}}{}_{\underline{\gamma}}\right)C_{\underline{\gamma\beta}}$$

$$= \bar{\Gamma}_{\underline{\beta}}{}^{\underline{\alpha}} := \frac{i}{3!}\varepsilon_{abc}\left(\Gamma_{\underline{abc}}u_{\underline{a}}{}^a u_{\underline{b}}{}^b u_{\underline{c}}{}^c\right)_{\underline{\beta}}{}^{\underline{\alpha}}. \tag{204}$$

Equation (149) in the case of the M2-brane splits into

对于 M2 膜的情形，式 (149) 可分解为

$$\mathcal{D}v_{\underline{\beta}}^{\alpha q}=\frac{1}{2}\Omega_{a}{}^{i}\gamma^{a}_{\alpha\beta}\gamma^{i}_{q\dot{p}}v_{\underline{\beta}\alpha\dot{p}},\ \mathcal{D}v_{\underline{\beta}\alpha\dot{q}}=-\frac{1}{2}v_{\underline{\beta}}^{\beta p}\Omega_{a}{}^{i}\gamma^{a}_{\alpha\beta}\gamma^{i}_{p\dot{q}}. \tag{205}$$

Appendix B: SO(1,5) × SO(5) Invariant Representation for $D = 11$ Dirac Matrices and M5-Brane Lorentz Harmonics

附录 B:SO(1,5) × SO(5) 对 $D = 11$ 狄拉克矩阵和 M5 膜洛伦兹调和函数的不变表示

The following $SO(1,5)\times SO(5)$ invariant representations for the 11D Dirac matrices and the charge conjugation matrix are suitable for the description of the M5-brane in the superembedding approach:

以下给出的 11 维狄拉克矩阵与电荷共轭矩阵的 $SO(1,5)\times SO(5)$ 不变表示，适用于描述超嵌入方法中的 M5 膜:

$$(\Gamma^{\underline{a}})_{\underline{\alpha}}{}^{\underline{\beta}}\equiv\left(\Gamma^{a},\Gamma^{i}\right),\ a=0,1,\ldots,5,\ i=1,\ldots,5,$$

$$(\Gamma^{a})_{\underline{\alpha}}{}^{\underline{\beta}}=\begin{pmatrix}0 & -i\gamma^{a}_{\alpha\beta}\delta_{q}{}^{p}\\ i\tilde{\gamma}^{a\alpha\beta}\delta_{q}{}^{p} & 0\end{pmatrix},$$

$$(\Gamma^{i})_{\underline{\alpha}}{}^{\underline{\beta}}\equiv\left(\Gamma^{1},\ldots,\Gamma^{8}\right)=\begin{pmatrix}\left(\gamma^{i}C\right)_{q}{}^{p}\delta_{\alpha}{}^{\beta} & 0\\ 0 & -\left(\gamma^{i}C\right)_{q}{}^{p}\delta^{\alpha}{}_{\beta}\end{pmatrix}$$

$$C^{\underline{\alpha\beta}}=-C^{\underline{\beta\alpha}}=\begin{pmatrix}0 & -i\delta^{\alpha}{}_{\beta}C^{qp}\\ -i\delta_{\alpha}{}^{\beta}C^{qp} & 0\end{pmatrix},\ C_{\underline{\alpha\beta}}=\begin{pmatrix}0 & i\delta_{\alpha}{}^{\beta}C_{qp}\\ i\delta^{\alpha}{}_{\beta}C_{qp} & 0\end{pmatrix}.$$

(206)

The Spin $(1,5)\sim SU^{*}(4)$ Clebsch-Gordan coefficients (Pauli-like matrices) are antisymmetric 4×4 matrices:

Spin $(1,5)\sim SU^{*}(4)$ 克莱布希-高登系数 (类泡利矩阵) 是反对称 4×4 矩阵:

$$\gamma^{a}_{\alpha\beta}=-\gamma^{a}_{\beta\alpha}=\gamma^{a}_{[\alpha\beta]},\ \tilde{\gamma}_{a}^{\alpha\beta}=-\tilde{\gamma}_{a}^{\beta\alpha}=\tilde{\gamma}_{a}^{[\alpha\beta]},\ \alpha=1,2,3,4. \tag{207}$$

They have the following properties:

它们具有如下性质:

$$\left(\gamma^{(a}\tilde{\gamma}^{b)}\right)_{\alpha}{}^{\beta}=\eta^{ab}\delta_{\alpha}{}^{\beta},\ \eta^{ab}=\mathrm{diag}\left(+,-,-,-,-,-\right),\ \tilde{\gamma}_{a}^{\alpha\beta}=\frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}\gamma_{a\gamma\delta}$$

$$\gamma^a_{\alpha\beta}\tilde{\gamma}_a^{\gamma\delta} = -4\delta_{[\alpha}{}^{\gamma}\delta_{\beta]}{}^{\delta},\ \gamma^a_{\alpha\beta}\gamma_{a\gamma\delta} = -2\varepsilon_{\alpha\beta\gamma\delta},\ \gamma^{abcdef}{}_{\alpha}{}^{\beta} = \varepsilon^{abcdef}\delta_{\alpha}{}^{\beta}$$

$$\gamma^{abc}{}_{\alpha\beta} = \gamma^{abc}{}_{(\alpha\beta)} = -\frac{1}{3!}\varepsilon^{abcdef}\gamma_{def\,\alpha\beta},\ \tilde{\gamma}^{abc\alpha\beta} = \tilde{\gamma}^{abc(\alpha\beta)} = \frac{1}{3!}\varepsilon^{abcdef}\gamma^{\alpha\beta}_{def}.$$

(208)

The matrices (207) are pseudo-real in the sense that the conjugate matrices $\gamma^{a*}_{\dot\alpha\dot\beta} := \left(\gamma^a_{\alpha\beta}\right)^*$ are expressed through $\gamma^a_{\alpha\beta}$ with the use of a matrix $B_\alpha{}^{\dot\alpha}$ [193] obeying $BB^* = -I$:

式 (207) 的矩阵是拟实的，即共轭矩阵 $\gamma^{a*}_{\dot\alpha\dot\beta} := \left(\gamma^a_{\alpha\beta}\right)^*$ 可通过满足 $BB^* = -I$：的矩阵 $B_\alpha{}^{\dot\alpha}$ 由 $\gamma^a_{\alpha\beta}$ 表出 [193]

$$\left(B\gamma^{a*}B^T\right) := B_\alpha{}^{\dot\alpha}\gamma^{a*}_{\dot\alpha\dot\beta}B_\beta{}^{\dot\beta} = \gamma^a_{\alpha\beta},\ \left(B^{*T}\tilde{\gamma}^{a*}B^*\right) := B^*{}_{\dot\alpha}{}^{\alpha}\tilde{\gamma}^{a*\dot\alpha\dot\beta}B_\beta{}^{\dot\beta} = \gamma^a_{\alpha\beta},$$

$$B_\alpha{}^{\dot\beta}B^*{}_{\dot\beta}{}^{\beta} = -\delta_\alpha{}^{\beta}$$

(209)

The properties of the Spin(5) $\sim USp(4)$ Clebsch-Gordan coefficients (Pauli-like matrices) γ^i_{qr} and $\tilde{\gamma}^{iqr}$, and the charge conjugation matrices C_{qr} and C^{qr} $(i,j = 1,\dots,5, q,p,r,s = 1,\dots,4)$ are

Spin(5) $\sim$ $USp(4)$ 克莱布希-高登系数 (类泡利矩阵) γ^i_{qr} 与 $\tilde{\gamma}^{iqr}$ ，以及电荷共轭矩阵 C_{qr} 与 C^{qr} $(i,j = 1,\dots,5, q,p,r,s = 1,\dots,4)$ 的性质为

$$\gamma^i\tilde{\gamma}^j + \gamma^j\tilde{\gamma}^i = 2\delta^{ij}\delta_q{}^p,\ \gamma^i\tilde{\gamma}^j - \gamma^j\tilde{\gamma}^i =:\ 2\gamma^{ij}{}_q{}^p, \tag{210}$$

$$\gamma^i_{qp} = -\gamma^i_{pq} = -\left(\tilde{\gamma}^{iqp}\right)^* = \frac{1}{2}\varepsilon_{qprs}\tilde{\gamma}^{irs}, \tag{211}$$

$$C_{qp} = -C_{pq} = -(C^{qp})^* = \frac{1}{2}\varepsilon_{qprs}C^{rs},\ C_{qr}C^{rp} = \delta_q{}^p,$$

$$C\tilde{\gamma}^iC = -\gamma^i,\ C\gamma^iC = -\tilde{\gamma}^i, \tag{212}$$

$$\gamma^i_{qp}\tilde{\gamma}^{irs} = -4\delta_{[q}{}^r\delta_{p]}{}^s - C_{qp}C^{rs},\ \gamma^i_{qp}\gamma^i_{rs} = -2\varepsilon_{qprs} - C_{qp}C_{rs}. \tag{213}$$

The relations (128) and (129) between the Lorentz-harmonic variables adapted to the embedding of the M5-brane are

适配 M5 膜嵌入的洛伦兹调和变量之间满足关系式 (128) 与 (129):

$$u^a_{\underline{b}}\Gamma^{\underline{b}}_{\underline{\alpha\beta}} = v^{\alpha q}_{\underline{\alpha}}(\gamma_a)_{\alpha\beta}v^{\beta p}_{\underline{\beta}}C_{qp} - v_{\underline{\alpha}\alpha}{}^{q}\tilde{\gamma}^{\alpha\beta}_a v_{\underline{\beta}\beta}{}^{p}C_{qp}, \tag{214}$$

$$u^i_{\underline{b}}\Gamma^{\underline{b}}_{\underline{\alpha\beta}} = 2iv_{(\underline{\alpha}|}{}^{\alpha q}\gamma^i_{qp}v_{|\underline{\beta})\alpha}{}^{p}, \tag{215}$$

The worldvolume covariant derivatives of the M5-brane Lorentz spinor harmonics have the following form:

M5 膜洛伦兹旋量调和的世界体积协变导数形式如下:

$$\mathcal{D}v_{\underline{\alpha}}{}^{\alpha q} = \frac{i}{2}\Omega^{ai} v_{\underline{\alpha}\beta}{}^{p} \tilde{\gamma}_a^{\beta\alpha} (\gamma^i C)_p{}^q, \; \mathcal{D}v_{\alpha q}{}^{\underline{\alpha}} = \frac{i}{2}\Omega^{ai} \gamma_{a\alpha\beta} (\gamma^i C)_q{}^p v_p^{\beta\underline{\alpha}}, \tag{216}$$

$$\mathcal{D}v_{\underline{\alpha}\alpha}{}^{q} = -\frac{i}{2}\Omega^{ai} v_{\underline{\alpha}}^{\beta p} \tilde{\gamma}_{a\beta\alpha} (\gamma^i C)_p{}^q, \; \mathcal{D}v_q{}^{\alpha\underline{\alpha}} = -\frac{i}{2}\Omega^{ai} \tilde{\gamma}_a^{\alpha\beta} (\gamma^i C)_q{}^p v_{\beta p}{}^{\underline{\alpha}}. \tag{217}$$

Cross-References

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- A Lightning Introduction to String Theory

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D Covariant Superspace Approaches to $\mathcal{N} = 2$ Supergravity

$\mathcal{N} = 2$ 超引力的 D 协变超空间方法

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